WIR 1

Wir 1: 12.1 to 12.3

Section 12.1

Problem 1. Find the center and radius of the sphere $x^2 + y^2 + z^2 + 4x - 2y - 8z = 5$. Does this sphere intersect the xz plane? If so, what is the intersection?

$$x^2+4x+4+y^2-2y+1+z^2-8z+16 = 5+4+1+16$$

$$(x+2)^2+(y-1)^2+(z-4)^2=26$$

$C(-2,1,4)$ $\quad r=\sqrt{26}$

$y=0$

$$(x+2)^2+1+(z-4)^2=26$$

yes! $\quad (x+2)^2+(z-4)^2=25$

circle

$C(-2,0,4)$, $r=5$

$x^2+y^2+z^2=1$

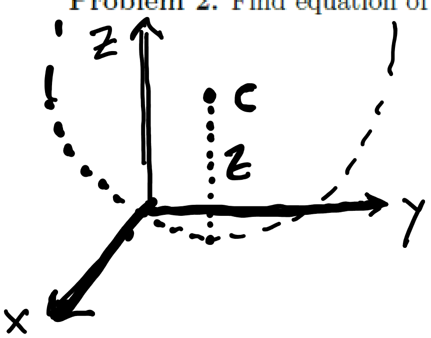
$z=5$

$x^2+y^2+25=1$

$\quad x^2+y^2=-24$

no intersection

Problem 2. Find equation of the sphere with center $(1, 2, 5)$ that touches the xy plane.

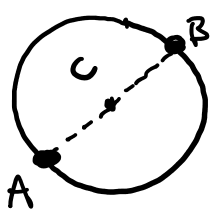


$$(x-1)^2+(y-2)^2+(z-5)^2=5^2$$

$$x^2-2x+1+y^2-4y+4+z^2-10z+\cancel{25}=\cancel{25}$$

$$x^2+y^2+z^2-2x-4y-10z=-5$$

Problem 3. Find the equation of the sphere if one of their diameters has endpoints A $(5, 1, 5)$ and B $(7, 3, 9)$.



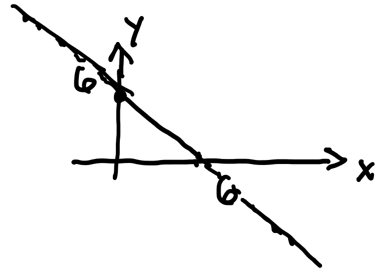
Need C = midpoint: $\left(\frac{5+7}{2}, \frac{1+3}{2}, \frac{5+9}{2}\right) = (6,2,7)\ C$

$|AB| = \sqrt{(7-5)^2+(3-1)^2+(9-5)^2} = \sqrt{4+4+16} = \sqrt{24} = 2\sqrt{6}$ = diam.

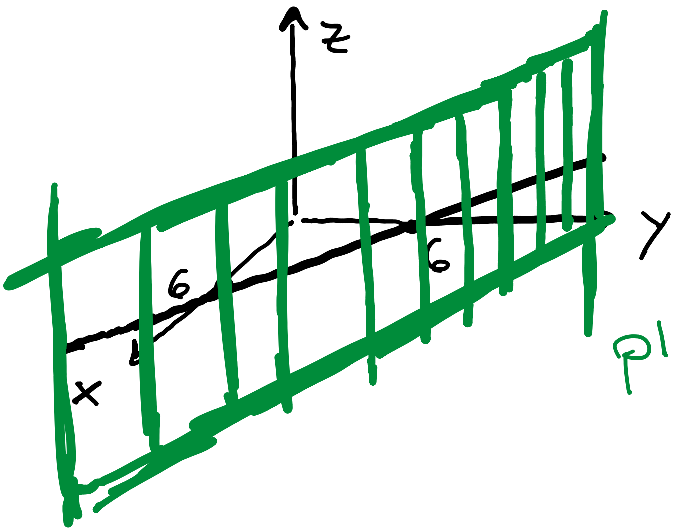
radius $= \frac{\cancel{2}\sqrt{6}}{\cancel{2}} = \sqrt{6}$

$$(x-6)^2+(y-2)^2+(z-7)^2=6$$

$(x-6) + (y-2) + (z-4) = 6$

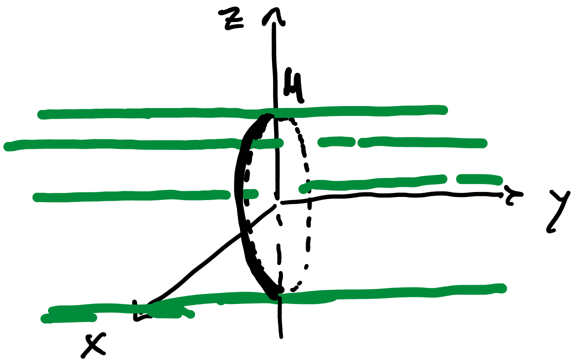


Problem 4. What does $y = 6 - x$ represent in $\mathbb{R}^3$?



plane

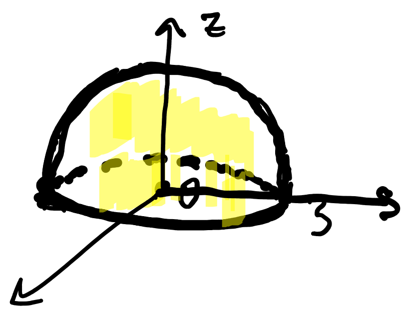
Problem 5. What does $x^2 + z^2 = 16$ represent in $\mathbb{R}^3$?



cylinder
y-axis
r=4.

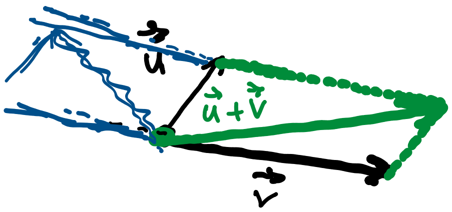
Problem 6. Write a set of inequalities that describes the solid upper hemisphere $x^2+y^2+z^2 = 9$.

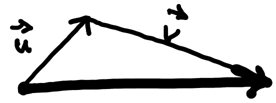
$$\begin{cases} x^2+y^2+z^2 \leq 9 \\ z \geq 0 \end{cases}$$



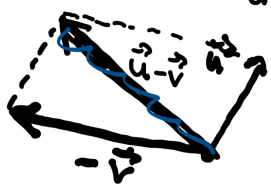
Section 12.2

Problem 7. Give a graphical interpretation of vector sum and vector difference.





$\vec{u}-\vec{v} = \vec{u}+(-\vec{v})$



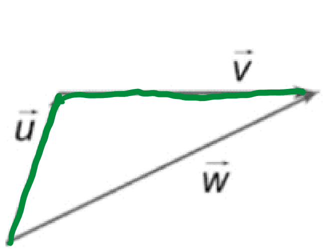
Problem 8. Given $\mathbf{a} = \langle -7, 1, 2\rangle$ and $\mathbf{b} = \langle 5, -1, 1\rangle$, find a unit vector in the direction of $\mathbf{a}+2\mathbf{b}$.

$\vec{a}+2\vec{b} = \langle -7, 1, 2\rangle + \langle 10, -2, 2\rangle = \langle 3, -1, 4\rangle = \vec{v}$

$\vec{u} = \dfrac{\vec{v}}{|\vec{v}|} = \dfrac{1}{\sqrt{26}}\langle 3, -1, 4\rangle = \left\langle \dfrac{3}{\sqrt{26}}, \dfrac{-1}{\sqrt{26}}, \dfrac{4}{\sqrt{26}} \right\rangle$

$|\vec{v}| = \sqrt{9+1+16} = \sqrt{26}$

Problem 9. For the picture seen below, write $\mathbf{v}$ in terms of $\mathbf{u}$ and $\mathbf{w}$.



$\vec{u}+\vec{v} = \vec{w}$

$\vec{v} = \vec{w}-\vec{u}$

Section 12.3

Problem 10. Compute $\mathbf{a}\cdot\mathbf{b}$ if

a.) $\mathbf{a} = \langle 4, 5, -1\rangle$ and $\mathbf{b} = \langle 2, 1, 3\rangle$. $\vec{a}\cdot\vec{b} = 8+5-3 = 10$

b.) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and $\theta = 120°$. $\vec{a}\cdot\vec{b} = |\vec{a}||\vec{b}|\cos\theta = 2\cdot 5\left(-\frac{1}{2}\right) = -5$

c.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and $\mathbf{a}$ is perpendicular to $\mathbf{b}$. $\vec{a}\cdot\vec{b} = 0$

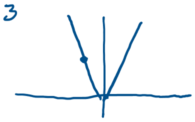
b.) $|\mathbf{a}| = 2$, $|\mathbf{b}| = 5$ and $\theta = 120°$. $\vec a\cdot\vec b = |\vec a||\vec b|\cos\theta = \ldots$

c.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and a is perpendicular to b. $\vec a\cdot\vec b = 0$

d.) $|\mathbf{a}| = 6$, $|\mathbf{b}| = 4$ and a is parallel to b. $\vec a\cdot\vec b = 6\cdot 4(\cos 0) = 24$

$\cos 120° = -\frac{1}{2}$

$2\frac{\pi}{3}$



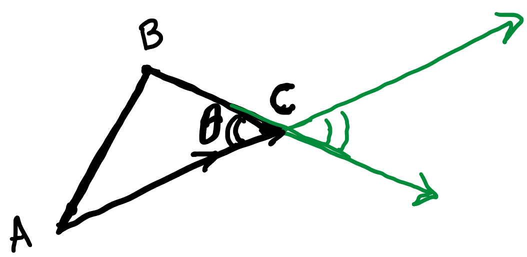
$\vec a \parallel \vec b$ means $\vec a$ and $\vec b$ are scalar multiples.

Problem 11. Are the vectors $-8\mathbf{i}+4\mathbf{j}+12\mathbf{k}$ and $6\mathbf{i}-3\mathbf{j}-9\mathbf{k}$ parallel, perpendicular, or neither?

$$-\frac{3}{4}\left(-8\hat{i}+4\hat{j}+12\hat{k}\right) = 6\hat{i}-3\hat{j}-9\hat{k}$$

Problem 12. The points $A(0,-1,6)$, $B(2,1,-3)$ and $C(5,4,2)$ form a triangle. Find $\angle C$.

$\angle BCA$



$$\cos\theta = \frac{\vec{AC}\cdot\vec{BC}}{|\vec{AC}||\vec{BC}|} =$$

$$\cos\theta = \frac{10}{\sqrt{66}\sqrt{43}}$$

$$\theta = \cos^{-1}\left(\frac{10}{\sqrt{66}\sqrt{43}}\right) \approx 1.382 \text{ rad.}$$

$\vec{AC} = C - A = \langle 5, 5, -4\rangle$

$\vec{BC} = C - B = \langle 3, 3, 5\rangle$

$|\vec{AC}| = \sqrt{25+25+16} = \sqrt{66}$

$|\vec{BC}| = \sqrt{9+9+25} = \sqrt{43}$

$\vec{AC}\cdot\vec{BC} = 15+15-20 = 10$

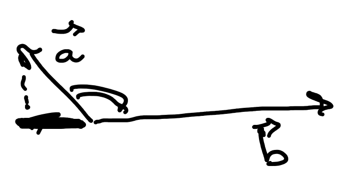
Problem 13. Find the vector and scalar projection of $\langle 1,2,5\rangle$ ($\vec a$) onto $\langle 0,7,4\rangle$ ($\vec b$).

$$\text{comp}_{\vec b}\vec a = |\vec a|\cos\theta = \frac{\vec a\cdot\vec b}{|\vec b|}$$

$$= \cancel{|\vec a|}\frac{\vec a\cdot\vec b}{\cancel{|\vec a|}|\vec b|}$$

$$\text{proj}_{\vec b}\vec a = \frac{\vec a\cdot\vec b}{|\vec b|}\frac{\vec b}{|\vec b|} = \left(\frac{\vec a\cdot\vec b}{|\vec b|^2}\right)\vec b$$

$$\text{comp}_{\vec b}\vec a = \frac{0+14+20}{\sqrt{0+49+16}} = \frac{34}{\sqrt{65}}$$



$$\text{comp}_{\vec{b}} = \sqrt{0+49+16} \qquad \sqrt{65}$$

$$\text{Proj}_{\vec{b}}\,\vec{a} = \left(\frac{34}{\sqrt{65}}\right) \langle 0, 7, 4 \rangle \frac{1}{\sqrt{65}} =$$

$$= \left\langle 0, \frac{238}{65}, \frac{136}{65} \right\rangle$$