Problem 1. What is the equation of the sphere centered at $(6, 4, 12)$ with radius 6? Describe the intersection of this sphere with the three coordinate planes.

$$(x-6)^2 + (y-4)^2 + (z-12)^2 = 36$$

xy-plane $z=0 \to (x-6)^2 + (y-4)^2 = 36 - 144$

$\to$ NO intersection

yz-plane $x=0$ $(y-4)^2 + (z-12)^2 = 36 - 36 = 0$

$(0, 4, 12)$ intersection

xz-plane $y=0$ $(x-6)^2 + (z-12)^2 = 36 - 16 = 20$

intersection = circle with center $(6, 0, 12)$ $r = \sqrt{20} = 2\sqrt{5}$.

Problem 2. Let $\mathbf{a} = \langle 1, 2, -1\rangle$ and $\mathbf{b} = \langle 2, -1, 2\rangle$. Find the vector projection of $\mathbf{b}$ onto $\mathbf{a}$, that is $\text{proj}_{\mathbf{a}}\mathbf{b}$.

$$\text{comp}_{\vec{a}}\vec{b} = \frac{\vec{a}\cdot\vec{b}}{|\vec{a}|} = \frac{\cancel{2} - \cancel{2} - 2}{\sqrt{1+4+1}} = \frac{-2}{\sqrt{6}}$$

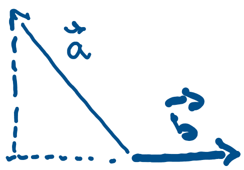
$$\text{proj}_{\vec{a}}\vec{b} = \left(\frac{-2}{\sqrt{6}}\right)\frac{\vec{a}}{|\vec{a}|} = \frac{-\cancel{2}^{1}}{\cancel{(\sqrt{6})^2}_{3}}\langle 1, 2, -1\rangle$$

Answer: $\langle -\frac{1}{3}, -\frac{2}{3}, +\frac{1}{3}\rangle$

Problem 3. Let $\mathbf{a} = \langle -2, 2, 1\rangle$. Find a vector $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ so that the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ equals -4, that is $\text{comp}_{\mathbf{a}}\mathbf{b} = -4$.

Problem 3. Let $\mathbf{a} = \langle -2, 2, 1\rangle$. Find a vector $\mathbf{b} = \langle b_1, b_2, b_3\rangle$ so that the scalar projection of $\mathbf{b}$ onto $\mathbf{a}$ equals -4, that is $\text{comp}_{\mathbf{a}}\mathbf{b} = -4$.

$$\text{comp}_{\vec a}\vec b = \frac{\vec a \cdot \vec b}{|\vec a| = \sqrt{9}} = \boxed{\frac{-2b_1 + 2b_2 + b_3}{3} = -4}$$



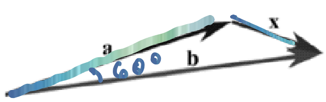
set $b_1 = 0 = b_2$

then $b_3 = -12$

$\langle 0, 0, -12\rangle$ ✓

$$\frac{b_3}{3} = -4 \to b_3 = 3(-4) = -12$$

Problem 4. Use the figure below to answer the questions that follow.



$\vec a + \vec x = \vec b \to \vec x = \vec b - \vec a$.

a.) Write $\mathbf{x}$ in terms of $\mathbf{a}$ and $\mathbf{b}$. ✓

b.) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $\mathbf{a} \cdot \mathbf{b}$.

c.) If the angle between $\mathbf{a}$ and $\mathbf{b}$ is $60°$, $|\mathbf{a}| = 7$, and $|\mathbf{b}| = 6$, find $|\mathbf{a} \times \mathbf{b}|$ and determine whether $\mathbf{a} \times \mathbf{b}$ is directed into or out of the page.

$$\vec a \cdot \vec b = |\vec a||\vec b|\cos\theta = 7\cdot 6\cos\frac{\pi}{3} = 7\cdot \cancel{6}^{3}\cancel{\tfrac{1}{2}} = 21$$

$$|\vec a \times \vec b| = |\vec a||\vec b|\sin\theta = 7\cdot 6\sin\frac{\pi}{3} = 7\cdot\cancel{6}^{3}\frac{\sqrt{3}}{\cancel{2}} = 21\sqrt{3}$$

Into the page —

Problem 5. Find a vector equation, a set of parametric equations, and symmetric equations for the line passing through the point $\underbrace{(-2,3,4)}_{P_0}$ that is parallel to the vector $\underbrace{\langle 1,-4,4\rangle}_{\vec v}$.

$$\boxed{\vec r = \vec r_0 + t\,\vec v}$$

$\vec r = \vec{OP}$, $\vec r_0 = \vec{OP_0}$

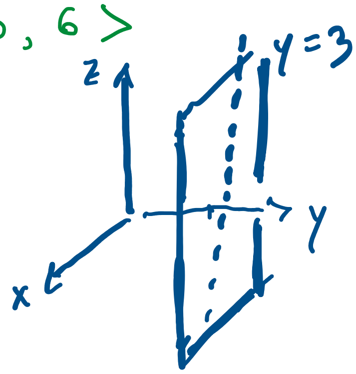
$\langle x, y, z\rangle = \langle -2, 3, 4\rangle + t\langle 1, -4, 4\rangle$ vector equ.

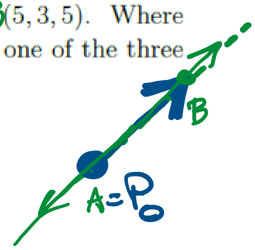
$$\begin{cases} x = -2 + t \\ y = 3 - 4t \\ z = 4 + 4t \end{cases}$$ param. equs

$$t = x+2 = \underbrace{\frac{y-3}{-4}}_{=\frac{3-y}{4}} = \frac{z-4}{4}$$ symm. equs.

Problem 6. Consider the line that passes through the points A $(4,3,-1)$ and B $(5,3,5)$. Where does this line intersect the three coordinate planes, and if it does not intersect one of the three coordinate planes, explain why not.

$\vec v = B - A = \vec{AB} = \langle 1, 0, 6\rangle$





$$\begin{cases} x = 4 + t \\ y = 3 + 0t \\ z = -1 + 6t \end{cases}$$

xz - no intersection

yz → set $x = 0$

$t = -4$

$(0, 3, -25)$

xy - $z = 0$ $\quad t = \frac{1}{6}$

$\left(\frac{25}{6}, 3, 0\right)$

↳ $4 + \frac{1}{6}$

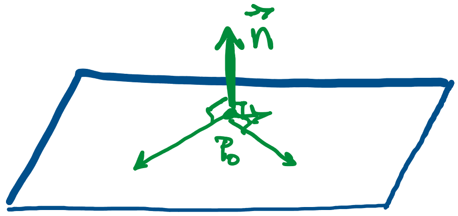
Problem 7. Find the equation of the plane that contains the point $\underbrace{(1,2,-5)}_{P_0}$ and is perpendicular to the vector $\langle -6, 4, -2\rangle = \vec n$

$\vec n \;|\; \vec{PD}$

P_0

$\vec{n} \perp \vec{P_0P}$

$\vec{n} \cdot \vec{P_0P} = 0$



$\langle -6, 4, -2 \rangle \cdot \langle x-1, y-2, z+5 \rangle = 0$

$-6(x-1) + 4(y-2) - 2(z+5) = 0$

$-6x + 4y - 2z - 12 = 0$ $6 - 8 - 10$

Problem 8. Find parametric equations for the line that passes through $\underbrace{(2, -1, 5)}_{P_0}$ and is

a.) parallel to the line $\frac{x+1}{3} = \frac{y-6}{4} = z$.

b.) perpendicular to the plane $8x - 11y = 2z + 6$.

$x + 1 = 3t$ $\begin{cases} x = -1 + 3t \\ y = 6 + 4t \\ z = 0 + t \end{cases}$

$\vec{v} = \langle 3, 4, 1 \rangle$

$\ell : \left[\begin{array}{l} x = 2 + 3t \\ y = -1 + 4t \\ z = 5 + t \end{array} \right.$ a

$8x - 11y - 2z - 6 = 0$

$\vec{n} = \langle 8, -11, -2 \rangle = \vec{v}$

$\left[\begin{array}{l} x = 2 + 8t \\ y = -1 - 11t \\ z = 5 - 2t \end{array} \right.$ b

Problem 9. Consider the triangle with vertices $\underbrace{P(1,0,1)}_{P_0}$, $Q(2,3,4)$ and $R(2,1,1)$.

a.) Find the angle at the vertex Q.

b.) Find the equation of the plane that passes through the points

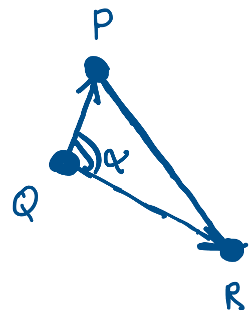
$\vec{QP} = P - Q = \langle -1, -3, -3 \rangle$

$\vec{QR} = R - Q = \langle 0, -2, -3 \rangle$

a

$$\alpha = \cos^{-1}\left(\frac{\vec{QP} \cdot \vec{QR}}{|\vec{QP}||\vec{QR}|}\right) =$$

$$= \cos^{-1}\left(\frac{0+6+9}{\sqrt{19}\sqrt{13}}\right) = \cos^{-1}\left(\frac{15}{\sqrt{19}\sqrt{13}}\right).$$



b

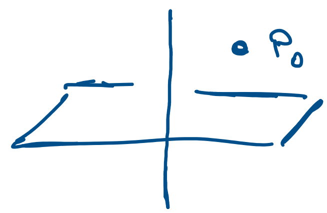
$$\vec{n} = \begin{vmatrix} i & j & k \\ 1 & 3 & 3 \\ 0 & 2 & 3 \end{vmatrix} = \langle 9-6, -(3-0), (2-0) \rangle = \langle 3, -3, 2 \rangle = \vec{n}$$

$$3(x-1) - 3(y-0) + 2(z-1) = 0$$

Problem 10. Find the equation of the $\boxed{\text{plane}}$ that passes through the point $\boxed{\overset{P_0}{(1,0,1)}}$ and

a.) is perpendicular to the line $x = 9 - t$, $y = 7 + 2t$, $z = t$.

b.) contains line $x = 9 - t$, $y = 7 + 2t$, $z = t$.



a

$\vec{n} = \langle -1, 2, 1 \rangle$

$-1(x-1) + 2(y-0) + 1(z-1) = 0$

$-x + 2y + z = 0$

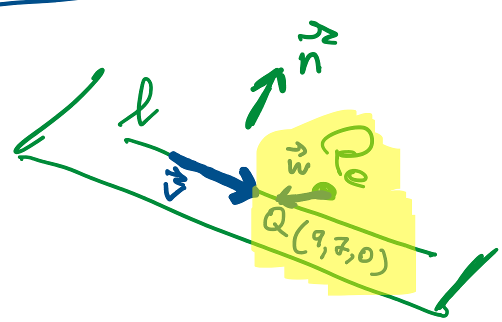
b

$\vec{n} = ?$

$\vec{n} = \vec{v} \times \vec{w}$

$\vec{v} = \langle -1, 2, 1 \rangle$

$w = \vec{P_0Q} = \langle 8, 7, -1 \rangle$



$$\vec{n} = \begin{vmatrix} i & j & k \\ -1 & 2 & 1 \\ 8 & 7 & -1 \end{vmatrix} = \langle -2-7, -(1-8), -7-16 \rangle =$$

$$= \langle -9, 7, -23 \rangle = \vec{n}$$

$9 + 14 - 23$ ✓

$-72 + 49 + 23$ ✓

$$-9(x-1) + 7(y-0) - 23(z-1) = 0$$

$-9(x-1)+7(y-0)-23(z-1)=0$

$-9x+7y-23z+32=0$

$9+1+1$
$4+1+9$
$6-1+6$

$\vec{n}_1=\langle 2,-1,3\rangle$

Problem 11. Consider the plane P_1 given by the equation $2x-y+3z=7$ and the plane P_2 given by the equation $3x+y+2z=3$. $\vec{n}_2=\langle 3,1,2\rangle$

a.) Find the angle between the planes. $\longrightarrow$ $\alpha=\cos^{-1}\left(\frac{11}{\sqrt{14}\sqrt{14}}\right)=\cos^{-1}\left(\frac{11}{14}\right)$

b.) Find a point, (x_0,y_0,z_0), that lies on both planes.

c.) Find a parametric equation for the line where the two planes intersect.

b

$2x-y+3z=7$
$3x+y+2z=3 \rightarrow y=3-3x-2z=3-6=-3$

$5x\ \cancel{0}+5z=10 \rightarrow x+z=2$ $\quad x=2-z$

$z=0,\ x=2,\ y=-3$ $\quad \boxed{(2,-3,0)\ P_0}$

$$\vec{v}=\begin{vmatrix} i & j & k \\ 2 & -1 & 3 \\ 3 & 1 & 2 \end{vmatrix}=\langle -2-3,\ -(4-9),\ 2+3\rangle=$$
$$=\langle -5,5,5\rangle$$

c

$$\begin{cases} x=2-5t \\ y=-3+5t \\ z=0+5t \end{cases}$$

Problem 12. Consider the lines $\mathbf{r}_1(t)=\langle 1,2,0\rangle+t\langle 2,-2,2\rangle$ and $\mathbf{r}_2(v)=\langle 3,0,2\rangle+v\langle -2,2,0\rangle$.

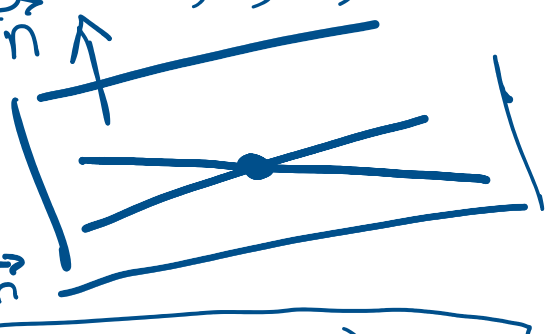
✓ a.) Find the point where the two lines intersect.

b.) Find an equation of the plane containing both of these lines.

$$\begin{cases} 1+2t=3-2v \rightarrow 3=3 \\ 2-2t=2v \rightarrow v=0 \\ 0+2t=2 \rightarrow t=1 \end{cases}$$

$P_0(3,0,2)$
$(3,0,2)$ ✓

$\vec{n}$



$$\vec{n}=\begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -1 & 1 \\ -1 & 1 & 0 \end{vmatrix}=\langle -1,-(1),0\rangle=$$
$$=\langle -1,-1,0\rangle=\vec{n}$$

$$\boxed{-1(x-3)-1(y-0)+0(z-2)=0}$$

$$\boxed{-1(x-3) - 1(y-0) + 0(z - \dots) = 0}$$

Problem 13. Let $\mathbf{r}(t) = \langle t^2, \frac{t-1}{t^2-1}, \frac{\sin t}{t} \rangle$.

a.) Find the domain of $\mathbf{r}(t)$.

b.) Find $\lim_{t\to 1} \mathbf{r}(\mathbf{t})$.

$t \neq \pm 1$

$t \neq 0$

$-1 \quad 0 \quad 1$

$(-\infty, -1) \cup (-1, 0) \cup (0, 1) \cup (1, +\infty)$

$\frac{\cancel{t-1}}{\cancel{(t-1)}(t+1)}$

$\langle 1, \frac{1}{2}, \frac{\sin 1}{1} \rangle$

domain of $\csc x$: $\frac{1}{\sin x}$

$(3, \pi) \cup (\pi, 2\pi) \cup (2\pi, 3\pi) \cup \dots$

$t \to 1$

Problem 14. Let $\mathbf{r}(t) = \langle \cos(t^2), \sin(t^2), t^2 \rangle$.

a.) Find the velocity and speed of the curve at time $t = \sqrt{\pi}$.

b.) Find $\mathbf{T}(\sqrt{\pi})$, the unit tangent vector, at $t = \sqrt{\pi}$.

c.) Find $\mathbf{a}(t)$, the acceleration vector, at time t.

d.) The length of the curve from the point $(1, 0, 0)$ to the point $(1, 0, 2\pi)$.

e.) The curvature of the curve traced out by $\mathbf{r}(t)$ when $t = \sqrt{\pi}$.

$t = \sqrt{2\pi}$

$\sqrt{4\pi + \pi^4} = \sqrt{8\pi}$

a) $\vec{v}(t) = \langle -\boxed{2t} \sin t^2, 2t \cos t^2, 2t \rangle \rightarrow \langle 0, -2\sqrt{\pi}, 2\sqrt{\pi} \rangle$

$|\vec{v}(t)| = \sqrt{4t^2(\sin^2(t^2) + \cos^2(t^2) + 1)} = \boxed{2t\sqrt{2}} \rightarrow 2\sqrt{2\pi}$

divide by $2\sqrt{2\pi}$

b) $\vec{T}(\sqrt{\pi}) = \langle 0, -\frac{1}{\sqrt{2}}, \frac{1}{\sqrt{2}} \rangle$

c) $\langle -2\sin t^2 - 2t \cdot 2t \cos t^2, 2\cos t^2 - 2t2t \sin t^2, 2 \rangle$

$t = \sqrt{\pi}$

$\rightarrow \langle -2\sin t^2 - 4t^2 \cos t^2, 2\cos t^2 - 4t^2 \sin t^2, 2 \rangle$

d) $\mathcal{L} = \int_a^b |\vec{v}(t)| \, dt = \int_0^{\sqrt{2\pi}} 2\sqrt{2}\, t \, dt = 2\sqrt{2} \frac{t^2}{2} \Big|_0^{\sqrt{2\pi}} = \sqrt{2} \cdot 2\pi$

e) $k = \frac{|\vec{r}\,' \times \vec{r}\,''|}{\dots} = \dots$

e) $k = \dfrac{|\vec{r}\,' \times \vec{r}\,''|}{|\vec{r}\,'|^3} = \dfrac{}{\sqrt{8\pi}^3}$

$$\begin{vmatrix} i & j & k \\ 0 & -2\sqrt{\pi} & 2\sqrt{\pi} \\ 4\pi & -2 & 2 \end{vmatrix} = \langle 0, -(-8\pi\sqrt{\pi}), -8\pi\sqrt{\pi} \rangle$$

Problem 15. Find parametric equations for the tangent line to the curve $x = 4\sqrt{t}$ (8), $y = t^2 - 10$ (6), $\boxed{1 = z = \frac{4}{t}}$ at $(8, 6, 1) = P(4)$

Screen clipping taken: 2/9/2021 8:27 PM

$\vec{v} = \langle 4\frac{1}{2\sqrt{t}}, 2t, -\frac{4}{t^2} \rangle$ @ $t=4$

$\boxed{\langle 1, 8, -\frac{1}{4} \rangle}$

$$\begin{cases} x = 8 + t \\ y = 6 + 8t \\ z = 1 - \frac{1}{4}t \end{cases}$$

Problem 16. If $\mathbf{r}'(t) = \langle t, e^t, te^{3t} \rangle$ and $\boxed{\mathbf{r}(0) = \langle 1, 3, 2 \rangle}$, find $\mathbf{r}(t)$.

Screen clipping taken: 2/9/2021 8:27 PM

$\vec{r} = \int \vec{r}\,'$

diff	integr
t	e^{3t}
1	$\frac{1}{3}e^{3t}$
0	$\frac{1}{9}e^{3t}$

$\vec{r}(t) = \langle \frac{t^2}{2} + c_1, e^t + c_2, \frac{t}{3}e^{3t} - \frac{1}{9}e^{3t} + c_3 \rangle$

$\vec{r}(0) = \langle c_1, 1 + c_2, -\frac{1}{9} + c_3 \rangle$

$\langle 1, 3, 2 \rangle$

$c_1 = 1 \quad c_2 = 2 \quad c_3 = 2 + \frac{1}{9} = \frac{19}{9}$

$C_1 = 1 \quad C_2 = 2 \quad C_3 = 2 + \frac{1}{9} = \frac{19}{9}$

Problem 17. Find $\int_0^1 \left(\frac{4t}{t^2+1}\mathbf{j} - \frac{1}{1+t^2}\mathbf{k} \right) dt$.

Screen clipping taken: 2/9/2021 8:27 PM

$$\left[2 \ln (t^2+1)\,\hat{j} - \tan^{-1} t\, \hat{k} \right]_0^1 =$$

$$= 2 \ln 5 - \cancel{2 \ln 1}\, \hat{j} - \left(\frac{\sqrt{2}}{2} - 0 \right) \hat{k}$$

Problem 18. Given the curves $\mathbf{r}_1(t) = \underbrace{\langle 3t, t^2, t^3 \rangle}_{t=0}$ and $\mathbf{r}_2(v) = \underbrace{\langle \sin v, \sin(2v), 6v \rangle}_{v=0}$ intersect at the origin, find the angle of intersection.

Screen clipping taken: 2/9/2021 8:27 PM

$\vec{r}_1{}'(t) = \langle 3, 2t, 3t^2 \rangle$

$r_2{}'(v) = \langle \cos v, 2\cos 2v, 6 \rangle$

$\langle 3, 0, 0 \rangle$

$\langle 1, 2, 6 \rangle$

$1+4+36$

$$\alpha = \cos^{-1}\left(\frac{3}{3\sqrt{41}} \right) = \cos^{-1}\left(\frac{1}{\sqrt{41}} \right)$$

Identify

Identify

$$\frac{4x^2+9y^2-36z^2=36}{36}$$

$$\frac{x^2}{9}+\frac{y^2}{4}-z^2=1$$

1-sheeted hyperboloid

$16x^2+4y^2+4z^2-64x+8y+16z=0$

$16(x^2-4x+4)+4(y^2+2y+1)+4(z^2+4z+4)=$

$=64+4+16$

$16(x-2)^2+4(y+1)^2+4(z+2)^2=84$

Ellipsoid

$-4x^2-8x-4+y^2+10y+25+16z^2+32z+16=$

$=0+37$

$-(2x+2)^2+(y+5)^2+(4z+4)^2=37$

1 sheeted hyperboloid

c V　　e IV　　d I

f III　　a II　　b VI