Wir 2

Tuesday, February 2, 2021 3:43 PM

Wir 2: 12.4, 12.5, 12.6

Section 12.4

1. Find the cross product of $\langle 1,1,3\rangle$ and $\langle -2,-1,5\rangle$ and find the area of the parallelogram determined by the two vectors.

$\vec{a} \downarrow$ $\vec{b} \downarrow$

$$\vec{a}\times\vec{b} = \begin{vmatrix} \hat{\imath} & \hat{\jmath} & \hat{k} \\ 1 & 1 & 3 \\ -2 & -1 & 5 \end{vmatrix} = \begin{vmatrix} 1 & 3 \\ -1 & 5 \end{vmatrix}\hat{\imath} - \begin{vmatrix} 1 & 3 \\ -2 & 5 \end{vmatrix}\hat{\jmath} + \begin{vmatrix} 1 & 1 \\ -2 & -1 \end{vmatrix}\hat{k} =$$

$$= (5+3)\hat{\imath} - (5+6)\hat{\jmath} + (-1+2)\hat{k} =$$

$$= 8\hat{\imath} - 11\hat{\jmath} + \hat{k}$$

$8 - 11 + 3 = 0$ ✓

$-16 + 11 + 5 = 0$

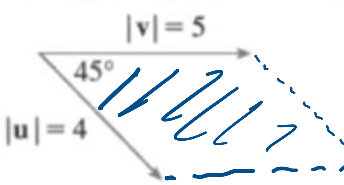
$$\text{Area} = |\vec{a}\times\vec{b}| = \sqrt{8^2 + (-11)^2 + 1^2} = \sqrt{64+121+1} = \boxed{\sqrt{186}}$$

121
64

$$\begin{array}{r|l} 186 & 2 \\ 93 & 3 \\ 31 & \end{array}$$

2. Find $|\vec{\mathbf{u}}\times\vec{\mathbf{v}}|$ and determine if $\vec{\mathbf{u}}\times\vec{\mathbf{v}}$ points in or out of the page.

out ✓



$$|\vec{u}\times\vec{v}| = |\vec{u}||\vec{v}|\sin\theta = 4\cdot 5\sin\frac{\pi}{4} =$$

$$= (\cancel{4}^{\,2})(5)\frac{\sqrt{2}}{\cancel{2}} = 10\sqrt{2} = \text{Area}$$

3. Find two unit vectors that are orthogonal to the plane that passes through the points $P(1,0,1)$, $Q(2,3,4)$ and $R(2,1,1)$.

$$\begin{matrix} & & \hat{\imath} & \hat{\jmath} & \hat{k} \\ \vec{PQ} = Q - P = & \langle & 1, & 3, & 3 & \rangle \\ \vec{PR} = R - P = & \langle & 1, & 1, & 0 & \rangle \end{matrix} = -3\hat{\imath} - (-3)\hat{\jmath} - 2\hat{k} =$$

$$= \langle -3, 3, -2\rangle \perp \text{plane}$$

$$\vec{u} = \frac{\langle -3, 3, -2\rangle}{|\langle -3, 3, -2\rangle|} =$$

$-3 + 9 - 6 = 0$ ✓

$-3 + 3 = 0$ ✓

$\frac{1}{\;}\langle -3,3,-2\rangle = \langle \frac{3}{\;}, \frac{3}{\;}, \frac{-2}{\;}\rangle$ ✓

$(k = 3, 1)\ \ \ \ 2 >$ $\qquad -3+3=0$

$$= \frac{1}{\sqrt{9+9+4}} \langle -3, 3, -2 \rangle = \left\langle -\frac{3}{\sqrt{22}}, \frac{3}{\sqrt{22}}, -\frac{2}{\sqrt{22}} \right\rangle ✓ \qquad \left\langle \frac{3}{\sqrt{22}}, -\frac{3}{\sqrt{22}}, \frac{2}{\sqrt{22}} \right\rangle ✓$$

4. Determine whether each expression is meaningful or meaningless (circle one). If so, state whether the expression is a vector or a scalar.

a.) $\vec{a} \cdot \vec{b}$	meaningful (vector or scalar)	meaningless
b.) $\vec{a} \times \vec{b}$	meaningful (vector or scalar)	meaningless
c.) $\vec{a} \bullet (\vec{b} \times \vec{c})$	meaningful (vector or scalar)	meaningless
d.) $\vec{a} \times (\vec{b} \times \vec{c})$	meaningful (vector or scalar)	meaningless
e.) $(\vec{a} \bullet \vec{b}) \times (\vec{c} \bullet \vec{d})$	meaningful (vector or scalar)	meaningless
f.) $(\vec{a} \times \vec{b}) \bullet (\vec{c} \times \vec{d})$	meaningful (vector or scalar)	meaningless
g.) $\vec{a} \times (\vec{b} \bullet \vec{c})$	meaningful (vector or scalar)	meaningless
h.) $\underbrace{\lvert\vec{a}\rvert}_{\text{scalar}}(\vec{b} \times \vec{c})$	meaningful (vector or scalar)	meaningless

Section 12.5

1. Find vector, parametric, and symmetric equations for the line through the point $\overbrace{(1, 0, -3)}^{P_0}$ and parallel to the vector $2\,\boldsymbol{i} - 4\,\boldsymbol{j} + 5\,\boldsymbol{k} = \vec{v}$

$$\begin{cases} x = x_0 + ta \\ y = y_0 + tb \\ z = z_0 + tc \end{cases}$$

if $\vec{v} = \langle a, b, c \rangle$

$$\begin{cases} x = 1 + 2t \\ y = 0 - 4t \\ z = -3 + 5t \end{cases}$$ param. equs.

$\langle x, y, z \rangle = \langle 1 + 2t, -4t, -3 + 5t \rangle$ vector eqn.

$$\begin{rcases} \cancel{2}t = \dfrac{x-1}{2} \\ t = \dfrac{y}{-4} \\ \cancel{5}t = \dfrac{z+3}{5} \end{rcases}$$

$$\frac{x-1}{2} = -\frac{y}{4} = \frac{z+3}{5}$$ symmetric equs

$$\vec{OP} = \vec{OP_0} + t\vec{v}$$

2. Find parametric ✓ and symmetric ✓ equations of the line through the points $\overbrace{(1, 2, 0)}^{A}$ and $\overbrace{(-5, 4, 2)}^{B}$.

$P_0(1, 2, 0)$ $\qquad \vec{v} = \vec{AB} = B - A = \langle -6, 2, 2 \rangle = \vec{v}$

$\{ x = 1 - 6t$ $\qquad$ param.

$$\begin{cases} x = 1 - 6t \\ y = 2 + 2t \\ z = 0 + 2t \end{cases}$$ param. equs.

$$\frac{x-1}{-6} = \boxed{\frac{y-2}{2} = \frac{z}{2} = \frac{1-x}{6}}$$ symmetric equs

#

3. Find parametric and symmetric equations of the line passing through the point $\overbrace{(-3, 5, 4)}^{P_0}$ and parallel to the line $\underbrace{x = 1 + 3t,\ y = -1 - 2t,\ z = 3 + t}_{\vec{v} = \langle 3, -2, 1 \rangle}$.

$$\begin{cases} x = -3 + 3t \\ y = 5 - 2t \\ z = 4 + t \end{cases}$$ Parametric equs.

$$\frac{x+3}{3} = \frac{y-5}{-2} = \boxed{z - 4 = \frac{5-y}{2} = \frac{x+3}{3}}$$

symmetric equs.

4. Find an equation of the plane through the point $\overbrace{(-4, 3, 1)}^{P_0}$ that is perpendicular to the vector $\vec{a} = -4\,\boldsymbol{i} + 7\,\boldsymbol{j} - 2\,\boldsymbol{k} = \vec{n}$

$P(x, y, z)$

$\vec{n} \cdot \overrightarrow{P_0P} = 0$ $\quad P_0P = P - P_0$

$\langle -4, 7, -2 \rangle \cdot \langle x+4, y-3, z-1 \rangle = 0$ $\quad -14$ $\quad -21$

$-4(x+4) + 7(y-3) - 2(z-1) = 0$ $\quad -16 - 21 + 2$

$$\boxed{-4x + 7y - 2z - 35 = 0}$$

5. Find an equation of the plane passing through the points $\overset{A}{(1, 2, -3)}$, $\overset{B}{(2, 3, 1)}$, and $\boxed{\overset{C}{(0, -2, -1)}}$. $\quad -12 - 3$

$\overrightarrow{AB} = B - A = \langle 1, 1, 4 \rangle$

$\vec{\ }$

$18(x-0) - 6(y+2) - 3(z+1) = 0$

$\boxed{18x - 6y - 3z - 15 = 0}$

$\vec{AB} = B - A = \langle 1, 1, 4 \rangle$

$\vec{AC} = C - A = \langle -1, -4, 2 \rangle$

$$\vec{n} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & 1 & 4 \\ -1 & -4 & 2 \end{vmatrix} = 18\hat{i} - 6\hat{j} - 3\hat{k} = \vec{n}$$

$18 - 6 - 12$ ✓

$-18 + 24 - 6$ ✓

$P_0\,(2, 3, 1)$

$18(x-2) - 6(y-3) - 3(z-1) = 0$

-36
$+18$
$+3$

$18(x-0) - 6(y+2) - 3(z+1) = 0$

$\boxed{18x - 6y - 3z - 15 = 0}$

$18x - 6y - 3z - 15 = 0$

36
21
-15

$\boxed{6x - 2y - z - 5 = 0}$

6. Determine whether the planes $3x + y - 4z = 3$ and $-9x - 3y + 12z = 4$ are orthogonal, parallel, or neither. Find the angle of intersection and the set of parametric equations for the line of intersection of the planes.

NO ✓

$\vec{n}_1 = \langle 3, 1, -4 \rangle$ $\quad$ $\vec{n}_2 = \langle -9, -3, 12 \rangle = \langle -3, -1, 4 \rangle$

orthogonal? $\vec{n}_1 \cdot \vec{n}_2 = -9 - 1 - 16 \neq 0$

parallel? $\frac{3}{-3} = \frac{1}{-1} = \frac{-4}{4} = -1$ $\quad$ PARALLEL.

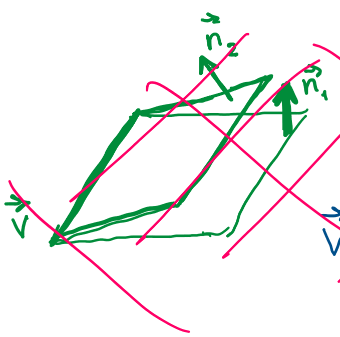
$$\theta = \cos^{-1}\frac{|\vec{n}_1 \cdot \vec{n}_2|}{|\vec{n}_1|\,|\vec{n}_2|} = \cos^{-1}\frac{|-27 - 3 - 48|}{\sqrt{9+1+16}\ \sqrt{81+9+144}} =$$

$$= \cos^{-1}\left(\frac{78}{\sqrt{26}\,\sqrt{234}}\right) = \cos^{-1}(1) = 0^\circ \qquad \cancel{\frac{3 \cdot 26}{\sqrt{26}\,3\sqrt{26}}} =$$

OR 0 radians.

234	2
117	3
39	3
13	13
1	1

$78 \,|\, 3$
26



$\vec{v} \perp \vec{n}_1, \vec{v} \perp \vec{n}_2$

$$\cancel{\vec{v} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 3 & 1 & -4 \\ -3 & -1 & 4 \end{vmatrix} = \langle 0}$$

no line of intersection because planes are parallel

7. Determine whether the planes $x - 3y + 6z = 4$ and $5x + y - z = 4$ are orthogonal, parallel, or neither. Find the angle of intersection and the set of parametric equations for the line of intersection of the planes.

$\vec{n}_1 = \langle 1, -3, 6 \rangle$ $\quad$ $\vec{n}_2 = \langle 5, 1, -1 \rangle$

$? \ \vec{n}_1 \perp \vec{n}_2 \ ?$ $\quad$ $5 - 3 - 6 \neq 0$ $\quad$ NOT orthogonal.

$-3 - 93 + 96 = 0$ ✓

of intersection of the planes. $\vec{n}_1 = \langle 1, -3, 6 \rangle$ $\vec{n}_2 = \langle 5, 1, -1 \rangle$

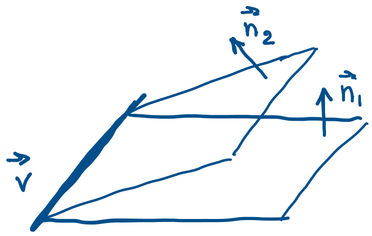
? $\vec{n}_1 \perp \vec{n}_2$? $5 - 3 - 6 \neq 0$ NOT orthogonal.

Ratios $\frac{1}{5}$ $\frac{-3}{1}$ $\frac{6}{-1}$ NOT PARALLEL

$-3 - 93 + 96 = 0$ ✓

$-15 + 31 - 16 = 0$ ✓

$$\vec{v} = \vec{n}_1 \times \vec{n}_2 = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ 1 & -3 & 6 \\ 5 & 1 & -1 \end{vmatrix} = \langle -3, +31, 16 \rangle = \vec{v}$$



Find a P_0 which is on both planes

$x - 3y + 6z = 4$

$[5x + y - z = 4](3)$

$15x + 3y - 3z = 12$

$16x + 0y + 3z = 16$

set $z = 0$ $x = 1$

$1 - 3y + 0 = 4 \to y = -1$

$P_0(1, -1, 0)$

$$\begin{cases} x = 1 - 3t \\ y = -1 + 31t \\ z = 0 + 16t \end{cases}$$

~~acute~~ planes

#

Angle of intersection:

$$\cos\theta = \frac{\vec{n}_1 \cdot \vec{n}_2}{|\vec{n}_1||\vec{n}_2|} = \frac{\langle 1, -3, 6 \rangle \cdot \langle 5, 1, -1 \rangle}{\sqrt{1+9+36}\ \sqrt{25+1+1}} = \frac{5-3-6}{\sqrt{46}\ \sqrt{27}}$$

$$\theta = \cos^{-1} \frac{|-4|}{\sqrt{46}\ 3\sqrt{3}} = \cos^{-1}\left(\frac{4}{3\sqrt{138}}\right) \checkmark$$

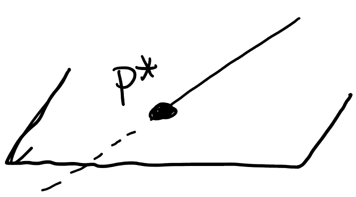
8. Find the point where the line $x = 1 + t$, $y = 2t$, and $z = -3t$ intersects the plane with equation $-4x + 2y - 4z = -2$.

$-4(1+t) + 2(2t) - 4(-3t) = -2$

$-4 - \cancel{4t} + \cancel{4t} + 12t = -2$

$12t = 2 \to t = \frac{1}{6}$

$P^*\left(\frac{7}{6}, \frac{1}{3}, -\frac{1}{2}\right)$



$1 + \frac{1}{6}$

#

9. Find the distance between point $\underbrace{(1, 2, 3)}_{P_1}$ and the plane with equation $2x - y + z = 4$.

$2x - y + z - 4 = 0$

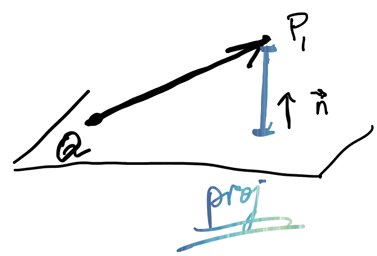
$$D = \frac{|2x_1 - y_1 + z_1 - 4|}{\sqrt{\ldots}} =$$

P_1

$$D = \frac{|2x_1 - y_1 + z_1 - 4|}{\sqrt{2^2+(-1)^2+1^2}} =$$

$$= \frac{|\cancel{2} - \cancel{2} + 3 - 4|}{\sqrt{4+1+1}} = \boxed{\frac{1}{\sqrt{6}}}$$

$2x - y + z - 4 = 0$



Section 12.6

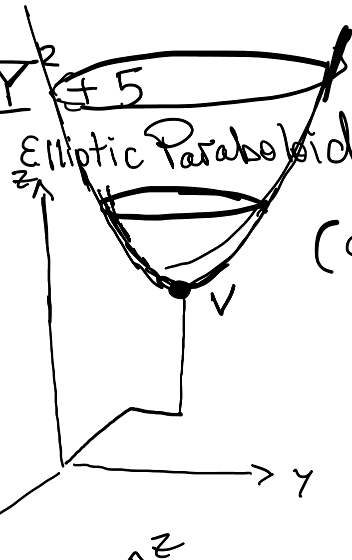
1. Identify and sketch the following quadric surfaces:

a) $z = \underbrace{(x+4)^2}_{} + \underbrace{(y-2)^2}_{} + 5.$ $\rightarrow$ $z = X^2 + Y^2 + 5$

$z - 5 = (x+4)^2 + (y-2)^2$

Elliptic Paraboloid

$V(-4, 2, 5)$

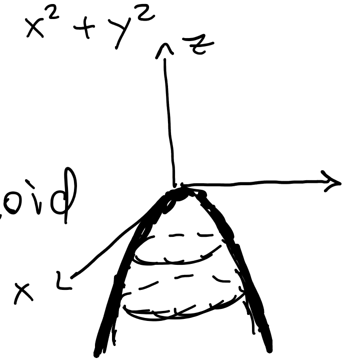


(a)

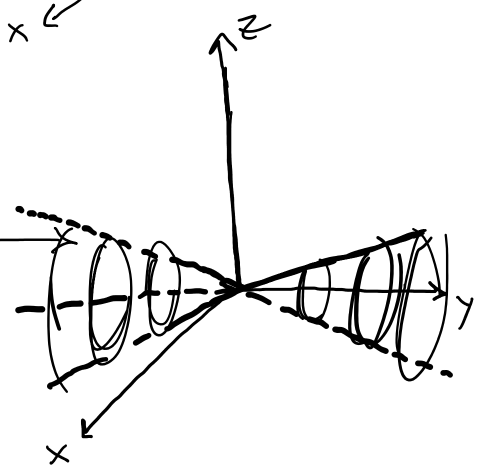
b) $z = -(x^2 + y^2)$

$\frac{z}{\underbrace{-1}_{c}} = x^2 + y^2$

Elliptic paraboloid



c) $y^2 = x^2 + z^2$



d) $x^2 + y^2 + z - 4x - 6y + 13 = 0.$

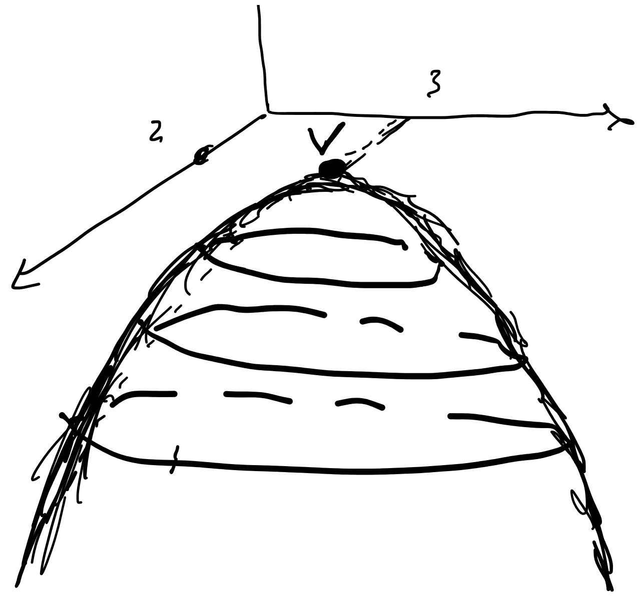
$x^2 - 4x + 4 \quad + y^2 - 6y + 9 \quad + z + \cancel{13} = 0 + \cancel{4+9}$

$(x-2)^2 + (y-3)^2 + z = 0$

$z = -\left[(x-2)^2 + (y-3)^2\right]$

Elliptic Paraboloid $\quad V(2, 3, 0)$

3


2
3
V