12.1 — 3D SPACE

Review

(a) The xyz axes should satisfy the ____________________ rule.

(b) In $\mathbb{R}^3$, the xz-coordinate plane is the plane containing the x and z axes. How can this plane be written in terms of an equation?

(c) To find a sphere, you need its center and radius. If (c_1, c_2, c_3) is the center and the radius is r, what is the standard formula for the equation of a sphere?

(d) What is the distance between the points (x_1, y_1, z_1) and (x_2, y_2, z_2)?

Exercise 1

Find an equation for the sphere with center at $(2, 1, -3)$ that just touches the plane $y = 5$.

Exercise 2

Let S be the sphere given by the equation $x^2 - 2x + y^2 + 8y + z^2 + 6z = 10$. Find an equation for the sphere centered at $(3, 2, 6)$ that just barely touches S.

Exercise 3

(a) Plot $(y-1)^2 + x^2 = 1$ in $\mathbb{R}^2$.

(b) Plot $(z-1)^2 + x^2 = 1$ in $\mathbb{R}^3$.

Exercise 4

In words, state what the following regions represent in $\mathbb{R}^3$.

(a) $x^2 + y^2 + z^2 \leq 1$ and $z > 0$.

(b) $(x-4)^2 + (z+2)^2 = 7$.

(c) $(x-4)^2 + (z+2)^2 = 7$ and $y = 4$.

Exercise 5

Let R be the region in $\mathbb{R}^3$ defined by $1 \leq (x-1)^2 + (y+2)^2 + z^2 \leq 9$. Let P be a plane. What are the possible shapes of $R \cap P$ (R intersected with P)?

12.2 — VECTORS

Review

(a) A vector has a ____________________ and a ____________________.

(b) If $\mathbf{v} = \langle v_1, v_2, v_3 \rangle$, then the length of $\mathbf{v}$ is

(c) A unit vector is a vector with ____________________.

(d) $\mathbf{i} = \langle$____, ____, ____$\rangle$ is the unit vector in the x-direction.

$\mathbf{j} = \langle$____, ____, ____$\rangle$ is the unit vector in the y-direction.

$\mathbf{k} = \langle$____, ____, ____$\rangle$ is the unit vector in the z-direction.

Exercise 6

An airplane currently (with respect to the ground) flying 200mph west, 100mph north, and 30mph up.

(a) How fast is the airplane going (with respect to the ground)?

(b) Assuming the airplane is oriented in its direction of travel, at what angle is the nose pointed up?

Exercise 7

Let $A = (5, 2, 7)$ and $B = (-2, 7, -3)$. Find the unit vector that points in the direction from A to B.

12.3 – THE DOT PRODUCT

Review

(a) The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is

which is also equal to

(b) Two vectors are perpendicular (or orthogonal) if the angle between them is $\pi/2$ (i.e., $90°$).

(c) Which vectors is the zero vector $\mathbf{0}$ orthogonal to?

(d) Two vectors are orthogonal if and only if their dot product is ____.

Exercise 8

Let $\mathbf{v} = a\mathbf{i} + 3\mathbf{j}$. For which values of a is $\mathbf{v}$ orthogonal to $\langle 1, 2, 5 \rangle$?

Exercise 9

Suppose $|\mathbf{a}| = 3$ and $|\mathbf{b}| = 2$.

(a) If $\mathbf{a} \cdot \mathbf{b} = 6$, what do we know about the orientations of $\mathbf{a}$ and $\mathbf{b}$?

(b) If $\mathbf{a} \cdot \mathbf{b} = -6$, what do we know about the orientations of $\mathbf{a}$ and $\mathbf{b}$?

Exercise 10

Find the angle between the lines $x + 2y = 7$ and $5x - y = 2$.