16.1 — VECTOR FIELDS

Exercise 1

Sketch the following vector fields.

(a) $\mathbf{F} = 0.3\,\mathbf{i} - 0.4\,\mathbf{j}$

(b) $\mathbf{F} = x\,\mathbf{j}$

(c) $\mathbf{F} = \nabla f$, where $f(x, y) = xy$.

16.2 — LINE INTEGRALS

Exercise 2

Compute the following line integrals.

(a) $\int_C xy^4 \, \mathrm{d}s$, where C is the right half of the circle $x^2 + y^2 = 16$.

(b) $\int_C x^2 \, \mathrm{d}x + y^2 \, \mathrm{d}y$, where C is the arc of the circle $x^2 + y^2 = 4$ from $(2, 0)$ to $(0, 2)$ followed by the line segment from $(0, 2)$ to $(4, 3)$.

Exercise 2 continued on next page…

(c) $\int_C y^2 z \, \mathrm{d}s$, where C is the line segment from $(3, 1, 2)$ to $(1, 2, 5)$.

(d) $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where $\mathbf{F} = \langle y, z, x \rangle$ and C is the curve given by $x = \sqrt{t}$, $y = t$, $z = t^2$, $1 \leq t \leq 4$.

16.3 — FUNDAMENTAL THEOREM FOR LINE INTEGRALS

Exercise 3

Check if the following vector fields are conservative or not. If they are conservative, find a potential function for the vector field.

(a) $\mathbf{F}(x, y) = (y^2 - 2x)\,\mathbf{i} + 2xy\,\mathbf{j}$

(b) $\mathbf{F}(x, y) = \langle ye^x + \sin(y), e^x + x\cos(y)\rangle$

Exercise 3 continued on next page…

(c) $\mathbf{F}(x,y) = \langle 2xy + y^{-2}, x^2 - 2xy^{-3} \rangle$ in the region where $y > 0$.

(d) $\mathbf{F}(x,y) = (\ln(y) + y/x)\mathbf{i} + (\ln(x) + x/y)\mathbf{j}$

Exercise 4

The following are all conservative vector fields. For each, find a potential function f and use it to evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$.

(a) $\mathbf{F}(x, y) = \langle 3 + 2xy^2, 2x^2y \rangle$. C is the arc of the hyperbola $y = 1/x$ from $(1, 1)$ to $(4, \frac{1}{4})$.

(b) $\mathbf{F}(x, y) = (1 + xy)e^{xy}\,\mathbf{i} + x^2e^{xy}\,\mathbf{j}$. C is given by $\mathbf{r}(t) = \langle \cos(t), 2\sin(t) \rangle$, $0 \leq t \leq \pi/2$.

Exercise 4 continued on next page...

(c) $\mathbf{F}(x, y, z) = \langle yze^{xz}, e^{xz}, xye^{xz} \rangle$. C is given by $\mathbf{r}(t) = \langle t^2 + 1, t + 1, t^2 \rangle$, $0 \leq t \leq 1$.

(d) $\mathbf{F}(x, y, z) = \langle \sin(y), x\cos(y) + \cos(z), -y\sin(z) \rangle$. C is given by $\mathbf{r}(t) = \langle \sin(t), t, 2t \rangle$, $0 \leq t \leq \pi/2$.

16.4 — GREEN'S THEOREM

Exercise 5

Use Green's Theorem to evaluate the following line integrals along the given positively oriented curve.

(a) $\int_C ye^x \, \mathrm{d}x + 2e^x \, \mathrm{d}y$, where C is the rectangle with vertices $(0,0)$, $(3,0)$, $(3,4)$, and $(0,4)$.

Exercise 5 continued on next page...

(b) $\int_C (y + e^{\sqrt{x}})\,\mathrm{d}x + (2x + \cos(y^2))\,\mathrm{d}y$, where C is the boundary of the region enclosed by the parabolas $y = x^2$ and $x = y^2$.

Exercise 6

Use Green's Theorem to evaluate the following. Be sure to check the orientation of the curve.

(a) $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where $\mathbf{F}(x, y) = \langle y\cos(x) - xy\sin(x), xy + x\cos(x) \rangle$ and C is the triangle from $(0, 0)$ to $(0, 4)$ to $(2, 0)$ to $(0, 0)$.

Exercise 6 continued on next page. . .

(b) $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where $\mathbf{F}(x, y) = \langle e^{-x} + y^2, e^{-y} + x^2 \rangle$ and C is the arc of the curve $y = \cos(x)$ from $(-\pi/2, 0)$ to $(\pi/2, 0)$ followed by the line segment from $(\pi/2, 0)$ to $(-\pi/2, 0)$.