FINAL EXAM REVIEW

Exercise 1

Sketch the vector field $\mathbf{F} = \langle x, y \rangle$.

Exercise 2

Compute $\int_C e^x \, \mathrm{d}x$, where C is the arc of the curve $x = y^3$ from $(-1, -1)$ to $(1, 1)$.

Exercise 3

Compute the line integral $\int_C xy^4 \, \mathrm{d}s$, where C is the right half of the circle $x^2 + y^2 = 16$.

Exercise 4

Compute the line integral $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where $\mathbf{F} = \langle x + y^2, xz, y + z \rangle$ and C is given by $\mathbf{r}(t) = \langle t^2, t^3, -2t \rangle$, $0 \leq t \leq 2$.

Exercise 5

Determine whether or not $\mathbf{F}(x, y) = \langle xy + y^2, x^2 + 2xy \rangle$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

Exercise 6

Determine whether or not $\mathbf{F}(x, y) = \langle ye^x + \sin(y), e^x + x\cos(y) \rangle$ is a conservative vector field. If it is, find a function f such that $\mathbf{F} = \nabla f$.

Exercise 7

Determine whether or not $\mathbf{F}(x, y, z) = \langle yze^{xz}, e^{xz}, xye^{xz} \rangle$ is a conservative vector field. If it is, find a potential function for the vector field $\mathbf{F}$. Evaluate $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where C is a curve from $(0, 0, 0)$ to $(1, 4, 2)$.

Exercise 8

Using Green's theorem, evaluate $\int_C y^3\,\mathrm{d}x - x^3\,\mathrm{d}y$, where C is the circle $x^2 + y^2 = 4$, oriented clockwise.

Exercise 9

Using Green's theorem, find the work done by the force field $\mathbf{F} = \langle x(x+y), xy^2 \rangle$ on a particle that moves from the origin along the x-axis to $(1,0)$, then along the line segment to $(0,1)$ and then back to the origin along the y-axis.

Exercise 10

Compute the curl and divergence of the vector field $\mathbf{F} = \sin(yz)\,\mathbf{i} + \sin(zx)\,\mathbf{j} + \sin(xy)\,\mathbf{k}$.

Exercise 11

Find a parametric representation of the following surfaces.

(a) The plane through the point $(1, 2, 1)$ that contains the vectors $\mathbf{i} - \mathbf{j}$ and $\mathbf{j} - \mathbf{k}$.

Exercise 11 continued on next page...

(b) The part of the cylinder $x^2 + z^2 = 9$ that lies above the xy-plane and between the planes $y = -4$ and $y = 4$.

(c) The part of the sphere $x^2 + y^2 + z^2 = 36$ that lies between the planes $z = 0$ and $z = 3\sqrt{3}$.

Exercise 12

Find the surface area of the following surfaces.

(a) The part of the plane $3x + 2y + z = 6$ that lies in the first octant.

Exercise 12 continued on next page...

(b) The part of the surface $x = z^2 + y$ that lies between the planes $y = 0$, $y = 2$, $z = 0$, and $z = 2$.

Exercise 13

Compute $\iint_S y^2 z^2 \, \mathrm{d}S$, where S is the part of the cone $y = \sqrt{x^2 + z^2}$ between the planes $y = 0$ and $y = 5$.

Exercise 14

Using Stokes' theorem, compute $\int_C \mathbf{F} \cdot \mathrm{d}\mathbf{r}$, where $\mathbf{F} = \langle xy, yz, zx \rangle$ and C is the boundary of the part of the paraboloid $z = 1 - x^2 - y^2$ in the first octant. (C is oriented counterclockwise as viewed from above.)

Exercise 15

Using Stokes' theorem, compute $\iint_S \operatorname{curl}(\mathbf{F}) \cdot \mathrm{d}\mathbf{S}$, where $\mathbf{F} = ze^y\,\mathbf{i} + x\cos(y)\,\mathbf{j} + xz\sin(y)\,\mathbf{k}$ and S is the hemisphere $x^2 + y^2 + z^2 = 16$, $y \geq 0$, oriented in the direction of the positive y-axis.

Exercise 16

Using the divergence theorem, compute the flux of $\mathbf{F} = \langle x^3 + y^3, y^3 + z^3, z^3 + x^3 \rangle$ across the surface S, where S is the sphere centered at the origin with radius 2.