12.3 — THE DOT PRODUCT (CONTINUED...)

Review

(a) The dot product of $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$ and $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ is

$$\mathbf{a} \cdot \mathbf{b} = a_1 b_1 + a_2 b_2 + a_3 b_3,$$

which is also equal to

$$|\mathbf{a}||\mathbf{b}| \sin \theta.$$

(b) Two vectors are perpendicular (or orthogonal) if the angle between them is $\pi/2$ (i.e., $90°$).

(c) The zero vector $\mathbf{0}$ is orthogonal to all vectors.

(d) Two vectors are **orthogonal** if and only if their dot product is 0.

(e) The **vector projection** of $\mathbf{b}$ onto $\mathbf{a}$, denoted $\text{proj}_{\mathbf{a}}\mathbf{b}$, is given by the following picture and formula:

(f) The **scalar projection** of $\mathbf{b}$ onto $\mathbf{a}$, denoted $\text{comp}_{\mathbf{a}}\mathbf{b}$, is given by the formula:

(g) The formula for **work** is

Exercise 1

Find the angles of the triangle formed by the points $A(1, 2, 3)$, $B(-1, 2, -4)$, and $C(2, 0, -1)$.

Exercise 2

Let E, F, and G be points in $\mathbb{R}^3$. Suppose $|\overrightarrow{EF}| = 3$ and $|\overrightarrow{EG}| = 4$. If $\overrightarrow{EF} \cdot \overrightarrow{EG} = 10$, then what is the angle $\angle FEG$? What is the angle between the vectors $\overrightarrow{EF}$ and $\overrightarrow{GE}$?

Exercise 3

Suppose the points P, Q, R form an acute triangle. Is $\text{comp}_{\overrightarrow{PQ}}\overrightarrow{QR}$ positive or negative? Draw an obtuse triangle where the opposite is true.

Exercise 4

Let $\mathbf{v} = (v_1, v_2, v_3)$. Then, we call v_1 the "x-component of $\mathbf{v}$". Explain why it makes sense that people often call $\text{comp}_{\mathbf{a}}\mathbf{v}$ the "$\mathbf{a}$-component of $\mathbf{v}$".

12.4 — THE CROSS PRODUCT

Review

(a) The cross product of $\mathbf{a} = (a_1, a_2, a_3)$ and $\mathbf{b} = (b_1, b_2, b_3)$ is

which is also equal to

(b) The result of the cross product is a ________________ that points in the direction given by the ______________________________.

(c) $\mathbf{a} \times \mathbf{b}$ is ____________________________ to both $\mathbf{a}$ and $\mathbf{b}$.

(d) Two vectors $\mathbf{a}$ and $\mathbf{b}$ are perpendicular if and only if

(e) $\mathbf{a} \times \mathbf{b} \neq \mathbf{b} \times \mathbf{a}$. However, we have

(f) The order in which we do cross products matters: $(\mathbf{a} \times \mathbf{b}) \times \mathbf{c} \neq \mathbf{a} \times (\mathbf{b} \times \mathbf{c})$.

(g) Cross products of the standard basis vectors:

$$\mathbf{i} \times \mathbf{j} = \qquad \mathbf{j} \times \mathbf{i} =$$
$$\mathbf{j} \times \mathbf{k} = \qquad \mathbf{k} \times \mathbf{j} =$$
$$\mathbf{k} \times \mathbf{i} = \qquad \mathbf{i} \times \mathbf{k} =$$

(h) The **area** of a parallelogram formed by the vectors $\mathbf{a}$ and $\mathbf{b}$ is

(i) The **volume** of a parallelepiped formed by the vectors $\mathbf{a}$, $\mathbf{b}$, and $\mathbf{c}$ is given by

(j) **Torque** is given by the formula

(k) The direction of the torque vector tells you

Exercise 5

Compute $\langle 1, 5, -2\rangle \times \langle -2, 1, 3\rangle$.

Exercise 6

Find the area of the triangle formed by the points $A(1, 2, 3)$, $B(-1, 2, -4)$, and $C(2, 0, -1)$.

Exercise 7

Find two unit vectors that are orthogonal to the plane that passes through the points $P(1, 0, 1)$, $Q(2, 3, 4)$, and $R(2, 1, 1)$.

12.5 – LINES AND PLANES

Review

(a) A **vector** (or **parametric**) equation for a line is of the form

(b) Taking the components of this equation, we obtain the ________________________ of the line:

$$x =$$

$$y =$$

$$z =$$

(c) The **symmetric equations** of a line are of the form

(d) __________ lines are lines that do not intersect and are not parallel.

(e) The standard form for a line segment is

Exercise 8

Find the parametric and symmetric equations for the line that goes through the points $(6, 2, -1)$ and $(2, -4, 1)$.

Exercise 9

For this problem, use the line you found in the previous exercise.

(a) Does the line go through the point $(-2, -8, 3)$?

(b) Where does the line intersect the plane $y = 100$?

Exercise 10

Find parametric equations for the line $\dfrac{x-3}{2} = \dfrac{y+6}{7} = \dfrac{z-2}{3}$.

Exercise 11

Determine if the following pair of lines is intersecting, parallel, or skew: $L_1(t) = \langle 1+t, -2-3t, 6+2t \rangle$ and $L_2(t) = \langle 4+t, -6+2t, 8-2t \rangle$.

Exercise 12

Find the parametric and symmetric equations for a line passing through $(5, -2, 1)$ that is parallel to the line $L(t) = \langle 6 - 2t, 5 + 4t, -2 - t \rangle$.

Exercise 13

Find a parametric equation for the line segment that starts at $A(3, -1, -2)$ and goes 5 units in the direction $\langle 2, 2, 1 \rangle$.