15.6 — TRIPLE INTEGRALS

Review

(a) We can take triple integrals by using an iterated integral. If $B = [a, b] \times [c, d] \times [r, s]$, then

$$\iint_D f(x, y)\,\mathrm{d}A = \int_a^b \int_c^d \int_r^s f(x, y)\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x.$$

(b) The order of integration can make the problem much easier or harder.

(c) The triple integral of 1 gives you the volume of the region of integration.

Exercise 1

Compute the following triple integrals.

(a) $\int_0^3 \int_0^x \int_{x-y}^{x+y} y\,\mathrm{d}z\,\mathrm{d}y\,\mathrm{d}x$

Exercise 1 continued on next page…

(b) $\int_0^1 \int_y^1 \int_0^{xy} e^{z/y} \, \mathrm{d}z \, \mathrm{d}x \, \mathrm{d}y$

Exercise 1 continued on next page...

(c) $\iiint_E \sin(y)\,\mathrm{d}V$, where E is the region below the plane $z = x$ and above the triangular region with vertices $(0, 0, 0)$, $(\pi, 0, 0)$, and $(0, \pi, 0)$.

Exercise 1 continued on next page...

(d) $\iiint_E xz\,\mathrm{d}V$, where E is the solid tetrahedron with vertices $(0,0,0)$, $(1,0,1)$, $(0,1,1)$, and $(0,0,1)$.

Exercise 2

Find the volume of the solid enclosed by the paraboloids $x = y^2 + z^2$ and $x = 4 - y^2 - z^2$.

Exercise 3

Find the volume of the solid enclosed by the cylinder $x^2+z^2=4$ and the planes $y=-1$ and $y+z=4$.

Exercise 4

Write the following integral in a few different orders of integration: $\int_0^1 \int_y^1 \int_0^y f(x, y, z)\,\mathrm{d}z\,\mathrm{d}x\,\mathrm{d}y$

15.2 — TRIPLE INTEGRALS IN CYLINDRICAL COORDINATES

Review

(a) Cylindrical coordinates are "polar coordinates plus a z direction."

(b) Conversion from rectangular to/from cylindrical coordinates:

$$x = r\cos\theta \qquad y = r\sin\theta \qquad z = z$$

$$r^2 = x^2 + y^2 \qquad \tan\theta = \frac{y}{x}$$

Exercise 5

Convert the following points from rectangular to cylindrical coordinates.

(a) $(x, y, z) = (-\sqrt{2}, \sqrt{2}, 1)$.

(b) $(x, y, z) = (2, 2, 2)$.

Exercise 6

Convert $(r, \theta, z) = (2, \pi/6, -1)$ to rectangular coordinates.

Exercise 7

Graph $r = 2$.

Exercise 8

Graph $\theta = \pi/3$.

Exercise 9

Sketch the solid $r^2 \leq z \leq 8 - r^2$.

Exercise 10

Sketch the solid $0 \leq \theta \leq \pi/2$, $r \leq z \leq 2$.

Exercise 11

For the following, set up the integral in cylindrical coordinates, but do not evaluate.

(a) $\iiint_E (x+y+z)\,\mathrm{d}V$, where E is the solid in the first octant under the paraboloid $z = 4 - x^2 - y^2$.

Exercise 11 continued on next page...

(b) $\iiint_E (x-y)\,\mathrm{d}V$, where E is the solid that lies between the cylinders $x^2+y^2=1$ and $x^2+y^2=16$, above the xy-plane, and below the plane $z=y+4$.