Wir 10: Sections 16.4, 16.5, 16.6, 16.7

Section 16.4

Green's Theorem: Let C be a positively oriented (counterclockwise) piecewise-smooth simple closed curve in the plane and let D be the region bounded by C. If P and Q have continuous partial derivatives on an open region that contains D, then

$$\oint_C P dx + Q dy = \iint_D \left(\frac{\partial Q}{\partial x} - \frac{\partial P}{\partial y} \right) \, dA$$

This says that the line integral over a simple closed curve C is equal to a double integral over the area of the region D the curve C encloses.

Note: We only use Green's theorem if we are on a **positively oriented closed** curve. If the curve is not positively oriented, then change the sign of the line integral. If not explicitly stated, assume counterclockwise orientation.

Problem Evaluate $\oint_C y^2\,dx + x\,dy$ where C is the triangular path from $(1,1)$ to $(3,1)$ to $(2,2)$ then back to $(1,1)$.

Problem ☐ Evaluate $\oint_C y^2\,dx + x^2\,dy$ where C is the boundary of the region bounded by the semicircle $y = \sqrt{4 - x^2}$ and the x axis. Assume positive orientation.

Problem ☐ Suppose a particle travels one revolution clockwise around the unit circle under the force field $\mathbf{F}(x, y) = \langle e^x - y^3, \cos(y) + x^3 \rangle$. Find the work done.

Section 16.5

Definition: The **del operator**, denoted by ∇, is defined as $\nabla = \left\langle \frac{\partial}{\partial x}, \frac{\partial}{\partial y}, \frac{\partial}{\partial z} \right\rangle$

Definition of curl and divergence:

Problem ☐ Find the divergence and curl of $\mathbf{F} = \langle xy, xz, xyz^2 \rangle$.

Theorem: If $\mathbf{F}$ is a vector field defined on all of $\Re^3$ whose component functions have continuous partial derivatives and curl $\mathbf{F} = \mathbf{0}$, then $\mathbf{F}$ is a conservative vector field. This gives us a way to determine whether a vector function on $\Re^3$ is conservative.

Problem ☐ If $\mathbf{F} = \langle x,\ e^y \sin z,\ e^y \cos z \rangle$, Find $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{r}(t) = \langle t^4, t, 2t^2 \rangle$, for $1 \le t \le 2$.

Problem ☐. If time permits, discuss what operaters make sense (similar to webassign section 16.5 problem 8)

Pb. 7

Evaluate $\iint\limits_S 2y\,dS$ where S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$.

Pb. 8

Evaluate $\displaystyle\iint_S xz\,dS$ where S is the portion of the sphere of radius 3 with $x \leq 0$, $y \geq 0$ and $z \geq 0$.

Pb. 9 SET UP ONLY. ***Note that all four surfaces are included.***

Evaluate $\iint\limits_S yz + 4xy\, dS$ where S is the surface of the solid bounded by $4x + 2y + z = 8$, $z = 0$, $y = 0$ and $x = 0$.

Answer to pb. 9:

$$\iint\limits_S yz + 4xy\, dS = \left(\frac{64\sqrt{21}}{3}\right) + (0) + \left(\frac{128}{3}\right) + \left(\frac{32}{3}\right) = \frac{64\sqrt{21}}{3} + \frac{160}{3} = \boxed{151.0949}$$

Pb. 10

Evaluate $\iint\limits_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = x^2\,\vec{i} + 2z\,\vec{j} - 3y\,\vec{k}$ and S is the portion of $y^2 + z^2 = 4$ between $x = 0$ and $x = 3 - z$ oriented outwards (i.e. away from the x-axis).

Pb. 11

Evaluate $\iint\limits_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = \vec{i} + z\,\vec{j} + 6x\,\vec{k}$ and S is the portion of the sphere of radius 3 with $x \leq 0$, $y \geq 0$ and $z \geq 0$ oriented inward (i.e. towards the origin).

Pb. 12 SET UP ONLY. ***Note that all three surfaces are included.***

Evaluate $\iint\limits_S \vec{F} \cdot d\vec{S}$ where $\vec{F} = yz\,\vec{i} + x\,\vec{j} + 3y^2\,\vec{k}$ and S is the surface of the solid bounded by $x^2 + y^2 = 4$, $z = x - 3$, and $z = x + 2$ with the negative orientation.

Note that all three surfaces of this solid are included in S.