

Week In Review #11 - Final Exam Review

NOTE: The final exam is cumulative. You need to review your OLD papers (quizzes, home works, exams, etc) for the final. This review will go over some problems which most students had trouble with during the semester, but it is not a complete review by itself.

Review Problems:

1. The world's economy has been expanding. In year 1994, the gross world product (total output in goods and services) was 35 trillion dollars and 45 trillion dollars in year 2000.
 - (a) If the gross has been growing linearly, find a formula for the gross world production.
 - (b) If the gross has been growing linearly, what is the average rate of change?
Interpret it.
 - (c) If the gross has been growing exponentially, find a formula for the gross world production.
 - (d) If the gross has been growing exponentially, what is the annual growth rate? Interpret it.
 - (e) If the gross has been growing exponentially, what is the continuous growth rate?

2. A picture supposedly painted by Vermeer (1632-1675) contains 96.5% of its Carbon-14 in 1998. The half-life of Carbon -14 is 5730 years. Determine whether or not the painting is a fake.

3. If the population of bacteria in a colony is increasing at a rate of 10% every 4 hours, what is the doubling time?

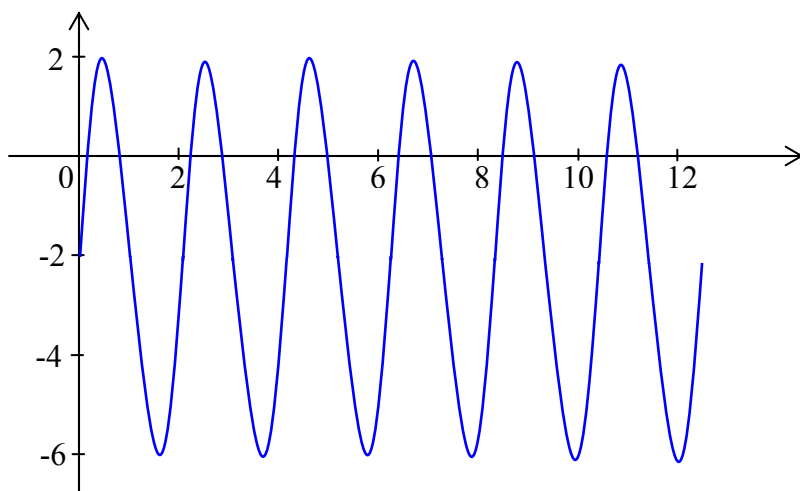
4. A cold virus spreads among a company of 400 people. The number of people, $N(t)$, who have had the virus by time t in hours since the virus started to spread can be approximated by a

function of the form $N(t) = \frac{400}{1 + 399e^{-0.4t}}$.

(a) Find $N(0)$ and interpret it.

(b) Approximately when is the virus spreading the fastest?

5. Find a possible equation for the function graphed below. Note the graph starts exactly at $x = 0$ and stops exactly when $x = 4\pi$.



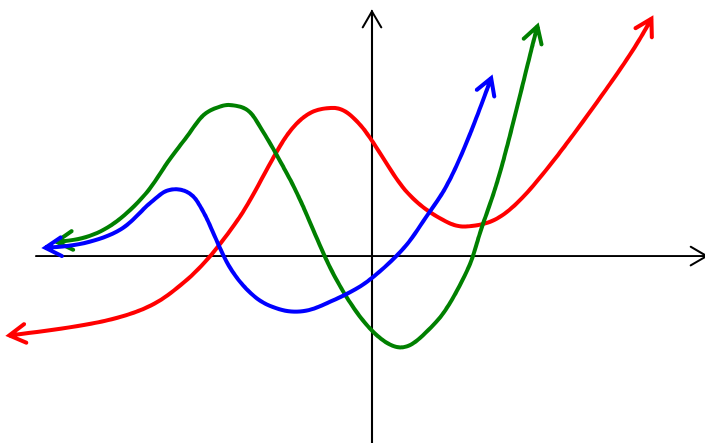
6. Given $f(x) = 0.05x^2 - 3.8x^5 + 4x^4 - 9x + 1.5$. Describe what type (even, odd, positive, negative, ...) of polynomial this will be. Without using a graphing calculator or computer, sketch a global view (in a large window) for the function.

7. (a) Find the average rate of change of $f(x) = (2 - x) \cdot 2^x$ between $x = 0$ and $x = 3$.

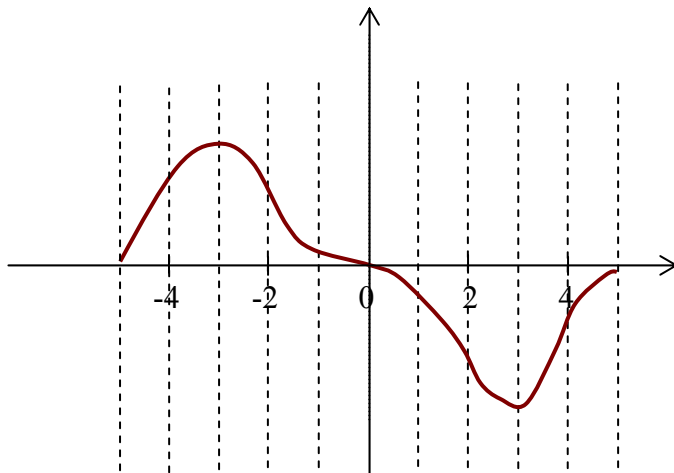
(b) Find the instantaneous rate of change of $f(x) = (2 - x) \cdot 2^x$ at $x = 0$.

(c) Find the equation of the tangent line to the function $f(x)$ at $x = 0$.

8. Identify each curve: one is $f(x)$, one is $f'(x)$ and one is $f''(x)$.



9. Using the graph below, answer the following questions:



- (a) If the graph above is of $g(x)$, on which interval is $g(x)$ decreasing?
- (b) If the graph above is of $g'(x)$, on which interval is $g(x)$ decreasing?
- (c) If the graph above is of $g'(x)$, on which interval are $g'(x)$ and $g''(x)$ both negative?
- (d) If the graph above is of $g'(x)$, on which interval is $g(x)$ concave down?
- (e) If the graph above is of $g''(x)$, on which interval is $g(x)$ concave down?
- (f) If the graph above is of $g(x)$, circle the smaller value for each of the following:

$g'(0)$ or $g(0)$ $g'(-3)$ or $g''(-3)$ $\frac{g(0) - g(-4)}{4}$ or $g'(2)$ $g'(4)$ or $g(4)$

- 10.** (a) Where is (are) the inflection point(s) of the function $f(x) = 3x^5 - 5x^4 - 30$ located?
(b) On what interval is $f(x) = 3x^5 - 5x^4 - 30$ both decreasing and concave up?

- 11.** Find the constants $a (\neq 0)$ and $b (\neq 0)$ in the function $f(x) = axe^{bx}$ such that $f(1/2) = 1$ and the function has a local maximum at $x = 1/2$.

12. In the following table the rate $R'(t)$ of sales for a firm in millions of dollars per month is given in monthly intervals. Assume $R(0) = 0$.

(a) Find the upper and lower estimates for the amount of sales during the five-month period.

(b) Interpret $R'(5)$.

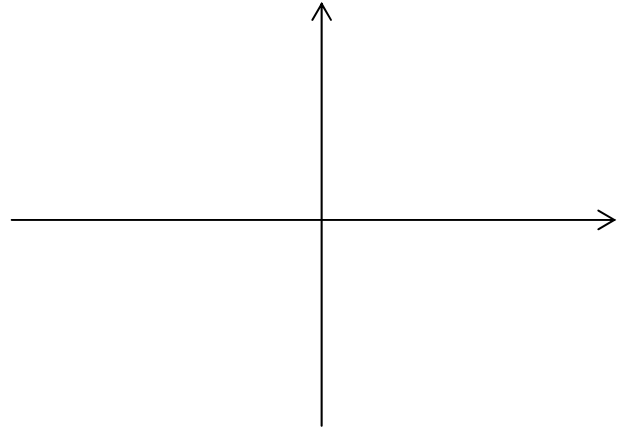
t	0	1	2	3	4	5
$R'(t)$	2	3	5	6	10	8

13. Find the area between the functions $y = x^3 - 5x - 1$ and $y = x^2$. Sketch the graph and shade the area between these two curves.

14. The population of bacteria in one cubic centimeter in the blood of a sick person can be modeled by $f(t) = 0.9t(t - 2)$, where t is in days since the person become ill. How fast is the population of bacteria changing one week after the onset of the illness?

15. Sketch a possible graph of a function that satisfies all of the given conditions.

- Vertical intercept at $y = -1$.
- Horizontal intercept at $x = 1$.
- Horizontal asymptote : $y = 1$
- Vertical asymptote : $x = -1$
- $f'(x) > 0$ for all x except $x = -1$.
- $f''(x) > 0$ on $(-\infty, -1)$ and $(0, 1)$.
- $f''(x) < 0$ on $(-1, 0)$ and $(1, \infty)$.



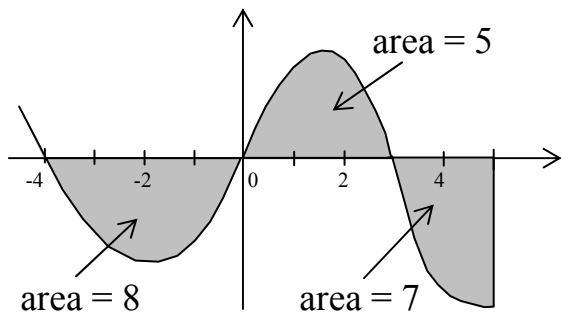
16. The population of Cleveland, Ohio from 1910 to 1970 is given by the table. The population is given in thousand of people. Let $t = 10$ represent 1910.

Year	1910	1920	1930	1950	1960	1970
Population	560.7	796.8	900.4	914.8	876.1	750.9

Round the answers to three decimal places.

- (a) Which of the three models would best fit the data: Linear, Quadratic, or Exponential?
- (b) Use your best-fit equation to predict the size of the population in 1980.

17. Given the graph of $F'(x)$ below with $F(-4) = 12$.



(a) Find all coordinates for all local maxima and local minima of $F(x)$.

(b) Which of following has the largest value?

$$F(0) - F(-4) \quad F(2) - F(-2) \quad F(4) - F(0)$$

(c) Sketch the graph of $F(x)$.

18. If $g'(x) = \frac{5 \cos(x^2 + 1)}{2x}$ and $g(1) = 5.5$, find $g(4)$.

19. Suppose oil is being pumped out of a well at a rate of $P'(t) = 2e^{-0.1t}$ with t measured in years and $P(t)$ in billions of barrels of oil. How long will it take to exhaust the well if the estimated capacity is 12 billion barrels?

20. A population in a confined space grows proportionally to the product of the current population, P , and the difference between the maximum population the environment can sustain, L , and the current population. Write a differential equation for the population, P , as a function of time.

21. Find the values of k such that $y = x^2 + k$ is a solution of the differential equation $2y - xy' = 10$.

22. Find the particular solution to the differential equation $4P' = P$ with the condition $P = 2e$, when $t = 4$.

Find the derivatives for following functions:

23. $f(x) = \frac{1}{2x} + 3 \cdot \sqrt[3]{x^5} + e^{4x} + \ln(2x) - 3 \cos x + \pi$

24. $y = \ln \sqrt{\frac{x^2(x-5)}{2e^x}}$

25. $s(x) = \left(\frac{1-2x^2}{\cos x} \right)^4$

26. $f(\theta) = 7 \sin(2\theta^2 + \theta + 2)e^{\cos \theta}$

Find indefinite integrals:

$$27. \int (1 - \sqrt[3]{x} + \frac{4}{x^2} + e^{-2x}) dx$$

$$28. \int \frac{3t^2 - 4}{(3t^3 - 12t + 15)^2} dt$$

$$29. \int \frac{5}{(2 - e^{-t})e^t} dt$$

$$30. \int \frac{\sin 5x}{\sqrt{1 + \cos 5x}} dx$$

31. $\int \frac{1}{4x \ln x^7} dx$

32. Find the exact value for $\int_2^4 \frac{\sqrt{x}}{x} dx$.

33. Find an antiderivative $F(t)$ with $F'(t) = (t^2 + 3)^2$ and $F(1) = 5$.

Answers:

1. (a) $G(t) = \frac{5}{3}t + 35$

(b) The slope is 1.67 trillion dollars per year. The gwp is increasing at an average rate of 1.67 trillion dollars per year since 1994.

(c) $G(t) = 35(1.0428)^t$

(d) The annual growth rate is 4.28%. The gwp is increasing 4.28% per year since 1994.

(e) The continuous growth rate is 4.19%.

2. The picture is about 295 years old and it was painted in 1703 ($1998 - 295 = 1703$), but Vermeer was already dead in 1703. Therefore the painting is a fake.

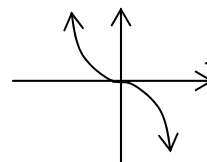
3. 29.1 hrs

4. (a) $N(0) = 1$ means that there was only one person who had the virus at $t = 0$.

(b) The virus is spreading fastest at $L/2$, so when $t = 15$ hours, the virus is spreading the fastest.

5. $y = 4\sin(3x) - 2$

6. $f(x)$ is an odd, negative, 5-degree polynomial. The global view will be



7. (a) $-10/3$

(b) $f'(0) = 2\ln 2 - 1$

(c) $y = (2\ln 2 - 1)x + 2$

8. Red: $f(x)$; Green: $f'(x)$; Blue: $f''(x)$.

9. (a) $(-3, 3)$ (b) $(0, 5)$ (c) $(0, 3)$

(d) $(-3, 3)$ (e) $(0, 5)$ (f) $g'(0), g''(-3), g'(2), g(4)$

10. (a) $x = 1$ (b) $(1, 4/3)$

11. $a = 2e$ $b = -2$

12.(a) 34, 24 millions

(b) $R'(5) = 8$ means that the company will expect an extra sales of 8 million dollars for the 6th month.

13. 16.393

14. 10.8 bacteria per cubic centimeter per day.

15. Graph.

16. (a) Quadratic

(b) 544 thousands

17. (a) Local min: $(0, 4)$ Local max: $(-4, 12), (3, 9)$

(b) $F(4) - F(0)$ is the largest.

(c) Graph.

18. 4.5418

19. 9.16 years

20. $\frac{dP}{dt} = kP(L - P)$

21. 5

22. $P = 2e^{t/4}$

23. $f'(x) = -\frac{1}{2x^2} + 5x^{2/3} + 4e^{4x} + \frac{1}{x} + 3 \sin x$

24. $\frac{1}{x} + \frac{1}{2(x-5)} - \frac{1}{2}$

25. $s(x) = \left[4 \cdot \left(\frac{1-2x^2}{\cos x} \right)^3 - \frac{4x \cos x + (1-2x^2) \sin x}{(\cos x)^2} \right]$

26. $7e^{\cos \theta} (\cos(2\theta^2 + \theta + 2) \cdot (4\theta + 1) - \sin(2\theta^2 + \theta + 2) \cdot \sin \theta)$

27. $x - \frac{3}{4}x^{4/3} - \frac{4}{x} - \frac{1}{2}e^{-2x} + C$

28. $-\frac{1}{3(3t^3 - 12t + 15)} + C$

29. $5 \ln |2 - e^{-t}| + C$

30. $-\frac{2}{5}\sqrt{1 + \cos 5x} + C$

31. $\frac{1}{28} \ln |\ln x^7| + C$

32. $2(2 - \sqrt{2})$

33. $F(t) = \frac{1}{5}t^5 + 2t^3 + 9t - 6.2$

If you find any mistakes, please let me know. Thanks! li-chen2@neo.tamu.edu