

Solutions to sample problems 1

1. $y - 7 = -1.4x$
2. $y - 2 = -2(x - 7)$
3. $y - 2 = .5(x - 7)$
4. $y = 2$
5. $y - 15 = \frac{5}{11}(x - 0)$
6. (a) $C(x) = 8x + 48,000$
 (b) \$40
 (c) $R(x) = 40x$
 (d) $P(x) = 32x - 48,000$
 (e) 1500 items
7. (a) equilibrium price \$6
 (b) equilibrium quantity 7
8. (a) $y = -.0864x + 11.8636$
 (b) see class notes.
 (c) 10.5676 million cows
 (d) 2014
 (e) The prediction is -3.6884 million cows. Note: negative answers means the model has failed.
 (f) 1933
9. There is more than one answer for this problem.

$$\left[\begin{array}{ccc|c} 1 & 0 & 2 & 7 \\ 0 & 1 & 5 & 8 \end{array} \right]$$
10. There is more than one answer for this problem.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 1 & 8 \\ 0 & 0 & 0 & 0 \end{array} \right]$$
11. There is more than one answer for this problem.

$$\left[\begin{array}{ccc|c} 1 & 0 & 0 & 6 \\ 0 & 1 & 0 & 5 \\ 0 & 0 & 0 & 8 \end{array} \right]$$
12. (a) **I)** x = the amount invested in high-risk stocks.
 y = the amount invested in medium-risk stocks.
 z = the amount invested in low-risk stocks.
II) $x + y + z = 300,000$
 $.16x + .10y + .04z = 33,000$
 $2x - y + 2z = 0$
III) $x = \$75,000$, $y = \$200,000$, and $z = \$25,000$

- (b) **I)** x = number of tank cars purchased with 6,000 gallon capacity
 y = number of tank cars purchased with 8,000 gallon capacity
 z = number of tank cars purchased with 18,000 gallon capacity

II) $x + y + z = 24$

$$6000x + 8000y + 18000z = 250000$$

III) Parametric solution:

$$x = -29 + 5z$$

$$y = 53 - 6z$$

$$z = \text{any number}$$

now to place restrictions on the parameter.

Since the number of cars has to be greater than or equal to zero.

$$\begin{array}{lll} x \geq 0 & y \geq 0 & z \geq 0 \\ -29 + 5z \geq 0 & 53 - 6z \geq 0 & \\ 5z \geq 29 & 53 \geq 6z & \\ z \geq 5.8 & 8.83333 \geq z & \\ & z \leq 8.83333 & \end{array}$$

Since the number of cars has to be less than or equal to 24.

$$\begin{array}{lll} x \leq 24 & y \leq 24 & z \leq 24 \\ -29 + 5z \leq 24 & 53 - 6z \leq 24 & \\ 5z \leq 53 & 29 \leq 6z & \\ z \leq 10.6 & 4.83333 \leq z & \\ & z \geq 4.83333 & \end{array}$$

Now using all of the above information at the same time, we see that $5.8 \leq z \leq 8.8333$. Since we can not buy a part of a tank car, z must be an integer so the only values of z that work are 6, 7, 8.

$$13. \left[\begin{array}{ccc|c} 3 & 6 & 15 & 9 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array} \right] \begin{array}{l} R_1(\frac{1}{3}) \rightarrow R_1 \\ \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array} \right] \begin{array}{l} R_2 + (-7)R_1 \rightarrow R_2 \\ 3R_3 + (-2)R_4 \rightarrow R_3 \\ \\ \end{array}$$

$$\left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ 0 & -2 & 4 & 4 \\ 0 & 18 & 3 & 10 \\ 3 & 0 & 6 & 1 \end{array} \right]$$

14. $x = 20$, $y = -11$, $u = 5$, and $z = -2$

$$15. K = \begin{bmatrix} 7 & -8 & 5 \\ -24.5 & 27 & -8.5 \\ 105 & -100 & 19 \end{bmatrix}$$

16. There is more than one solution for this problem. As long as matrix A and B are not square matrices and the number of rows in matrix B is equal to the number of columns in matrix A, you will have a solution.

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 3 & 4 & 2 \end{bmatrix}$$

$$B = \begin{bmatrix} 1 & 2 & 4 & 6 \\ 2 & 8 & 4 & 2 \\ 1 & 4 & 5 & 6 \end{bmatrix}$$

17. $D + C =$ not possible: not same dim.

$$D - 3B = \begin{bmatrix} -2 & 1 & -9 \\ -1 & -3 & -1 \end{bmatrix}$$

$$DC = \begin{bmatrix} 1 & -6 \\ 7 & 6 \end{bmatrix}$$

$DA =$ not possible: the number of rows in A is not equal to the number of cols. in D.

$$B + C^T = \begin{bmatrix} 2 & -1 & 7 \\ -2 & 4 & 0 \end{bmatrix}$$

B^{-1} not possible B is not square.

$$A^{-1} = \begin{bmatrix} 1 & 0 \\ -0.5 & -0.5 \end{bmatrix}$$

E^{-1} not possible, singular matrix.

$$18. (a) WP = \begin{bmatrix} 68.05 \\ 60.10 \end{bmatrix}$$

- (b) Each number represents the hourly rate for each crew. John's crew has an hourly rate of \$68.05 and Matt's crew has an hourly rate of \$60.10.

$$19. (a) x = -14, y = 39, z = -9$$

$$(b) x = -12, y = 37, z = -10$$

$$20. (a) A = \begin{bmatrix} 0.1 & 0.2 & 0.4 \\ 0.3 & 0.3 & 0.1 \\ 0.1 & 0.2 & 0.2 \end{bmatrix}$$

- (b) \$812.5 million of agriculture
\$568.75 million of manufacturing
\$343.75 million of transportation.

- (c) i. \$23 billion of agriculture is used internally
\$19 billion of manufacturing is used internally
\$15 billion of transportation is used internally

- ii. \$7 billion of agriculture
\$1 billion of manufacturing
\$25 billion of transportation