Sample Problems For Exam 1

Spring 2008 Compiled by Joe Kahlig

This collection of questions is intended to give an idea of different types of question that might be asked on the exam. This is not intended to represent an actual exam.
These questions cover chapters 1 and 2 in the Applied Finite Mathematics, 8^{th} edition by S. T. Tan.

Video solutions can be found at this link: http://www.math.tamu.edu/~kahlig/141WIRpage.html

1. Find the equation of the line with x-intercept of 5 and y-intercept of 7.

2. Find the equation of the line through the point $(7, 2)$ and parallel to $4x + 2y = 7$.

3. Find the equation of the line through the point $(7, 2)$ and perpendicular to $4x + 2y = 7$.

4. Find the equation of the line through the point$(7, 2)$ and parallel to $y = 5$.

5. As a person descends into the ocean, pressure increases linearly. The pressure is 15 pounds per square inch on the surface and 30 pounds per square inch 33 feet below the surface. If y is the pressure in pounds per square inch and x is the depth below the surface in feet, write an equation that expresses the pressure in terms of the depth.

6. Auto-time, a manufacturer of 24-variable timers, has a monthly fixed cost of \$48,000 and a production cost of \$8 for each timer manufactured.

   (a) Find the Cost function.
   (b) What is the selling price of the timers, if the company has a profit of \$112,000 when selling 5,000 timers.
   (c) Find the Revenue function.
   (d) Find the Profit function.
   (e) How many items should be sold to break even?

7. The supply function for a product is given by $3x - 11p + 45 = 0$ and the demand function for this product is given by $2x + 7p - 56 = 0$.

   (a) What is the equilibrium price?
   (b) What is the equilibrium quantity?

8. The number of milk cows in the United States is given in the table. (Source: US Dept. of Agriculture and from *Finite Mathematics: Practical Applications* by Johnson/Mowry).

Year	1970	1975	1980	1985	1990
Millions of Cows	12.091	11.220	10.758	10.777	10.153

   For This problem let the time start with zero in 1970.

   (a) Determine the equation of the least-squares (regression) line for this data.
   (b) Sketch a scatter diagram and the least-squares line for the data.
   (c) Predict the number of cows in the year 1985.
   (d) In what year would we expect to have 8 million milk cows?
   (e) Predict the number of cows in the year 2150.
   (f) In what year would we expect to have 15 million milk cows?

9. Give an example of a matrix in row-reduced form (reduced row-echelon form) that describes a system with an infinite number of solutions.

10. Give an example of a matrix in row-reduced form (reduce row-echelon form) with exactly one solution.

11. Give an example of a matrix in row-reduced form (reduce row-echelon form) with no solution.

12. For the next two word problems do the following.
I) Define the variables that are used in setting up the system of equations.
II) Set up the system of equations that represent this problem.
III) Solve for the solution.
IV) If the solutions is parametric, then tell what restrictions can be placed on the parameter. Also give three specific solutions.

(a) The management of a private investment club has a fund of \$300,000 earmarked for investment in stocks. To arrive at an acceptable overall level of risk, the stocks that management is considering have been classified into three categories: high-risk, medium-risk, and low-risk. Management estimates that high-risk stocks will have a rate of return of 16 percent per year; medium-risk stocks, 10 percent per year; and low-risk stocks, 4 percent per year. The investment in medium-risk stocks is to be twice the investment in stocks of the other two categories combined. If the investment goal is to have an average rate of return of 11 percent per year on the total investment, determine how much the club should invest in each type of stock.

(b) A chemical manufacturer wants to purchase a fleet of 24 railroad tank cars with a combined carrying capacity of 250,000 gallons. Tank cars with three different carrying capacities are available: 6,000 gallons, 8,000 gallons, and 18,000 gallons. How many of each type of tank car should be purchased?

13. Fill in the missing entries by performing the indicated row operations.

$$\left[\begin{array}{ccc|c} 3 & 6 & 15 & 9 \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array}\right] \quad R_1(\tfrac{1}{3}) \to R_1 \quad \left[\begin{array}{ccc|c} * & * & * & * \\ 7 & 12 & 39 & 25 \\ 2 & 6 & 5 & 4 \\ 3 & 0 & 6 & 1 \end{array}\right] \quad \begin{array}{l} R_2 + (-7)R_1 \to R_2 \\ 3R_3 + (-2)R_4 \to R_3 \end{array} \quad \left[\begin{array}{ccc|c} 1 & 2 & 5 & 3 \\ * & * & * & * \\ * & * & * & * \\ 3 & 0 & 6 & 1 \end{array}\right]$$

14. Solve for the variables x, y, z, and u. If this is not possible, then explain why

$$\begin{bmatrix} 1 & -1 & 0 \\ 0 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix} \begin{bmatrix} -2x & 0 \\ 3 & 4 \\ x+2 & 3 \end{bmatrix} - 3 \begin{bmatrix} y-1 & 2 \\ 1 & 2 \\ 4 & 2z+1 \end{bmatrix} = \begin{bmatrix} -7 & -2u \\ 0 & -2 \\ 8 & 10 \end{bmatrix}$$

15. Find the matrix K that makes the following true. If this is not possible, then explain why.

$$\begin{bmatrix} 0 & 8 & 1 \\ 7 & -6 & 0 \\ 3 & 0 & 1 \end{bmatrix} + \frac{1}{2} \begin{bmatrix} 2 & 0 & 0 \\ 5 & 2 & 0 \\ 6 & 6 & 1 \end{bmatrix} K = \begin{bmatrix} 7 & 0 & 6 \\ 0 & 1 & 4 \\ 3 & 7 & 0 \end{bmatrix}$$

16. Find a matrix A and a matrix B such that AB can be computed but BA can not be computed.

17. Use the following matrices for this problem. Compute the following operations. If it is not possible, then explain why.

$$A = \begin{bmatrix} 1 & 0 \\ -1 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 2 & 1 \end{bmatrix} \quad C = \begin{bmatrix} 1 & -2 \\ 0 & 2 \\ 4 & -1 \end{bmatrix} \quad D = \begin{bmatrix} 1 & -2 & 0 \\ -1 & 3 & 2 \end{bmatrix} \quad E = \begin{bmatrix} 1 & -1 & 3 \\ 0 & 1 & 0 \\ 1 & -2 & 3 \end{bmatrix}$$

$D + C =$ $\quad D - 3B =$ $\quad DC =$

$DA =$ $\quad B + C^T =$ $\quad B^{-1} =$

$A^{-1} =$ $\quad E^{-1} =$

18. John and Matt have a roofing company and they each have a crew that they work with to roof houses. John and Matt have given each worker a designation based on the number of years of experience. A worker who has less than one year of experience is classified as N. A worker with at least one and less than 3 years of experience is classifies as B. A worker with at least 3 and less than 7 years of experience is classified as E. A worker with more than 7 years of experience is classified as VE. The breakup of John and Matt's crews can be shown in this matrix W.

$$P = \begin{matrix} \text{N} \\ \text{B} \\ \text{E} \\ \text{VE} \end{matrix} \begin{bmatrix} 5.15 \\ 6.25 \\ 7.50 \\ 9.00 \end{bmatrix} \qquad W = \begin{matrix} \\ \text{John} \\ \text{Matt} \end{matrix} \begin{bmatrix} \text{N} & \text{B} & \text{E} & \text{VE} \\ 2 & 3 & 4 & 1 \\ 4 & 2 & 0 & 3 \end{bmatrix}$$

The matrix P represents the pay per hour of each worker designation.

(a) Find WP

(b) Explain the meaning of the entries in the matrix WP.

19. Solve the systems of equations by using inverses.

$$\begin{aligned} 2x + y + z &= b_1 \\ 5x + 2y + z &= b_2 \\ 3x + 2y + 4z &= b_3 \end{aligned}$$

(a) $b_1 = 2$, $b_2 = -1$, $b_3 = 0$

(b) $b_1 = 3$, $b_2 = 4$, $b_3 = -2$

20. An economy is based on three sectors: agriculture, manufacturing and transportation. Production of 1 unit of agriculture requires an input of 0.10 units from the agriculture sector, 0.30 units from the manufacturing sector, and 0.10 units from the transportation sector. Production of 1 unit of manufacturing requires an input of 0.20 units from the agriculture sector, 0.30 units from the manufacturing sector and 0.2 units from the transportation sector.Production of 1 unit of transportation requires an input of 0.4 units from the agriculture sector, 0.1 units from the manufacturing sector and 0.2 units from the transportation sector.

(a) Find the input-output matrix A.

(b) Find the output from each sector that is needed to satisfy a final demand of $480 million for agriculture, $120 million for manufacturing, and $80 million for transportation.

(c) If the economy has production of $30 billion for agriculture, $20 billion for manufacturing, and $40 billion for transportation, then

i. How much of this production is used in the production process of the economy?

ii. How much of this production is available for the demand of the consumers? (i.e. external demand)