# Section 12.2: Vectors

The term **vector** is used to represent a quantity that has both magnitude and direction. we denote a vector by putting a letter in boldface( **a**) or by putting an arrow above the letter ($\overrightarrow{a}$). The **zero vector**, denoted **0**, has length zero and is the only vector that does not have a specific direction.

**Definition:**A two dimensional vector has the form $\mathbf{a} = \langle a_1, a_2 \rangle$ and a three dimensional vector has the form $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, where $a_1$, $a_2$, and $a_3$ are real numbers and are called the components of the vector.

**Definition:** Given the points $A(x_1, y_1, z_1)$ and $B(x_2, y_2, z_2)$, the vector **a** with representation $\overrightarrow{AB}$ is

$$\mathbf{a} = \langle x_2 - x_1, y_2 - y_1, z_2 - z_1 \rangle$$

Example: For the points, $A(1, 2, 8)$ and $B(4, 7, 2)$, find $\overrightarrow{AB}$ and $\overrightarrow{BA}$.

$$\overrightarrow{AB} = \langle 3, 5, -6 \rangle$$

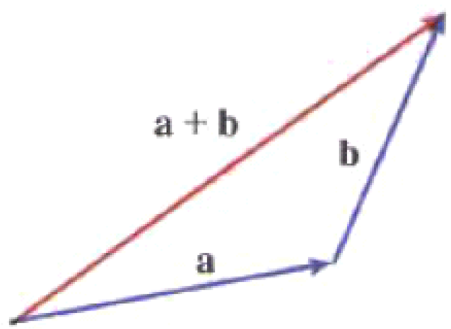
$$\overrightarrow{BA} = \langle -3, -5, 6 \rangle$$

**Definition:** Let $\mathbf{a} = \langle a_1, a_2, a_3 \rangle$, $\mathbf{b} = \langle b_1, b_2, b_3 \rangle$ and $c$ be a scalar, i.e. $c \in \Re$.

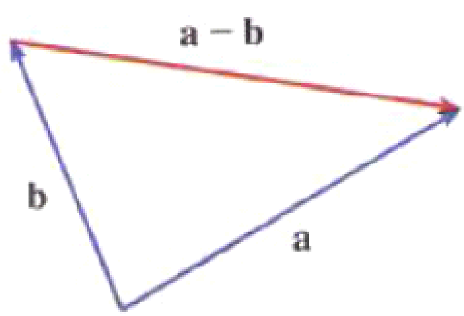
Scalar Multiplication: $c\mathbf{a} = c\langle a_1, a_2, a_3 \rangle = \langle ca_1, ca_2, ca_3 \rangle$

Length or magnitude of $\mathbf{a}$ is $|\mathbf{a}| = \sqrt{a_1^2 + a_2^2 + a_3^2}$

Vector Addition: $\mathbf{a} + \mathbf{b} = \langle a_1 + b_1, a_2 + b_2, a_3 + b_3 \rangle$



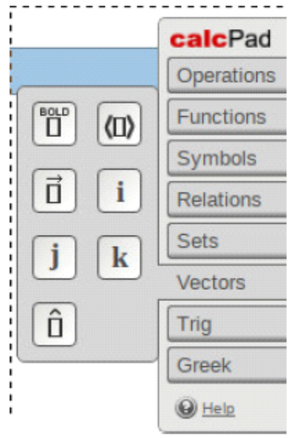

Vector Subtraction: $\mathbf{a} - \mathbf{b} = \langle a_1 - b_1, a_2 - b_2, a_3 - b_3 \rangle$



**Definition:** Two vectors are parallel if one vector is a scalar multiple of the other. i.e. there exists a $c \in \Re$ such that $c\mathbf{a} = \mathbf{b}$.

$$\langle 3, 4, 5 \rangle \qquad \langle 9, 12, 15 \rangle$$

**Definition:** A vector of length 1 is called a **unit vector**. The vectors $\mathbf{i} = \langle 1, 0, 0 \rangle$, $\mathbf{j} = \langle 0, 1, 0 \rangle$ and $\mathbf{k} = \langle 0, 0, 1 \rangle$ are called the standard basis vectors for $\Re^3$.




$$\langle 3,4,5 \rangle = 3\hat{i} + 4\hat{j} + 5\hat{k}$$

To find a unit vector in the same direction as $\mathbf{a}$, divide vector $\mathbf{a}$ by its magnitude. This process is called normalizing $\mathbf{a}$.

$$a = \langle 1, 2, 3 \rangle$$

$$|a| = \sqrt{1^2 + 2^2 + 3^2}$$

$$= \sqrt{1 + 4 + 9}$$

$$= \sqrt{14}$$

$$\text{unit}: \frac{1}{\sqrt{14}} \langle 1, 2, 3 \rangle$$

$$= \left\langle \frac{1}{\sqrt{14}}, \frac{2}{\sqrt{14}}, \frac{3}{\sqrt{14}} \right\rangle$$

Example: Find the following using the vectors: $\mathbf{a} = \langle 1, 2, 4\rangle$ and $\mathbf{c} = \langle 2, -4, 1\rangle$.

A) $3\mathbf{a} - 2\mathbf{c} = \langle 3, 6, 12\rangle - \langle 4, -8, 2\rangle$

$= \langle -1, 14, 10\rangle$

B) Find a vector of length 3 in the opposite direction of $\mathbf{a}$.

$|a| = \sqrt{1^2+2^2+4^2} = \sqrt{1+4+16}$

$= \sqrt{21}$

unit: $\frac{1}{\sqrt{21}}\langle 1, 2, 4\rangle$

Answer $= \frac{-3}{\sqrt{21}}\langle 1, 2, 4\rangle = \left\langle \frac{-3}{\sqrt{21}}, \frac{-6}{\sqrt{21}}, \frac{-12}{\sqrt{21}}\right\rangle$