

Section 14.1: Functions of Several Variables

$$y = f(x) = x^2$$

Definition: A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $z = f(x, y)$. The set D is the domain of f and its range is the set of values that f takes on, that is $\{f(x, y) | (x, y) \in D\}$.

Definition: If f is a function of two variables with domain D , the graph of f is the set $\{(x, y, z) \in \mathbb{R}^3 | z = f(x, y), (x, y) \in D\}$.

Example: Find the domain and sketch the graph of the function. What is the range?

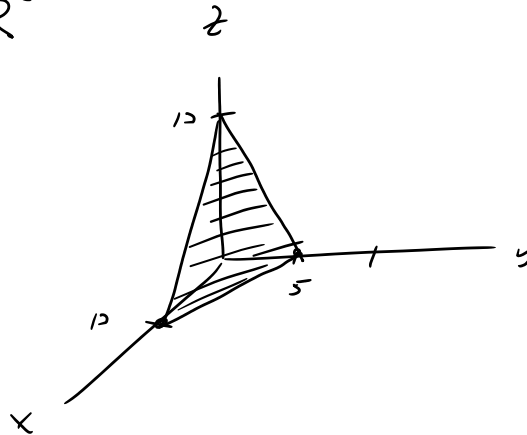
(a) $f(x, y) = 10 - x - 2y \iff z = 10 - x - 2y \leftarrow \text{plane}$

Domain is the xy plane

Range = all Real #s.

ie $\mathbb{R} \times \mathbb{R}$ or \mathbb{R}^2

x	y	z
0	0	10
0	5	0
10	0	0



$$\begin{aligned} 0 &= 10 - x - 2y \\ 2y &= 10 \\ y &= 5 \end{aligned}$$

$$\begin{aligned} 0 &= 10 - x - 0 \\ x &= 10 \end{aligned}$$

$$\{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 \leq 16\}$$

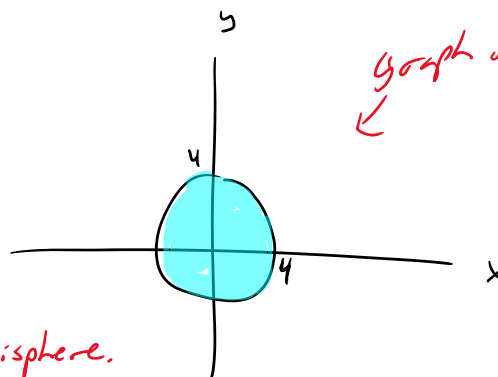
$$(b) f(x, y) = \sqrt{16 - x^2 - y^2}$$

$$D = \{(x, y) \mid x^2 + y^2 \leq 16\}$$

$$16 - x^2 - y^2 \geq 0$$

$$16 \geq x^2 + y^2$$

$$x^2 + y^2 \leq 16$$

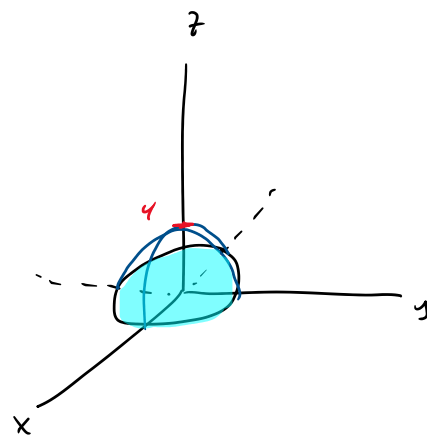


$$z = \sqrt{16 - x^2 - y^2} \quad \leftarrow \text{upper hemisphere.}$$

$$z^2 = 16 - x^2 - y^2$$

$$x^2 + y^2 + z^2 = 16 \quad \leftarrow \text{sphere.}$$

$$\text{Range } \{z \mid 0 \leq z \leq 4\}$$



Example: Find and sketch the domain.

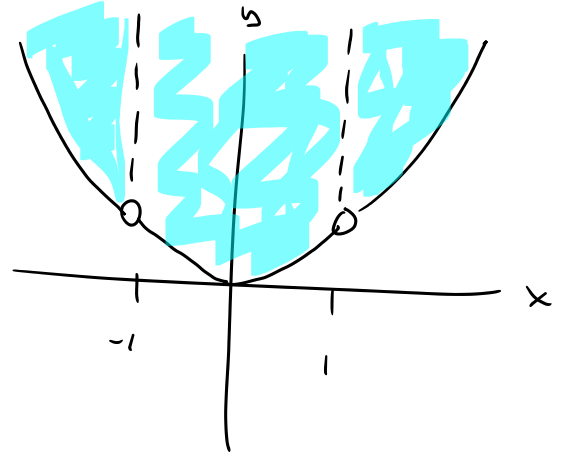
$$(a) z = \frac{\sqrt{y-x^2}}{1-x^2}$$

$$x \neq \pm 1$$

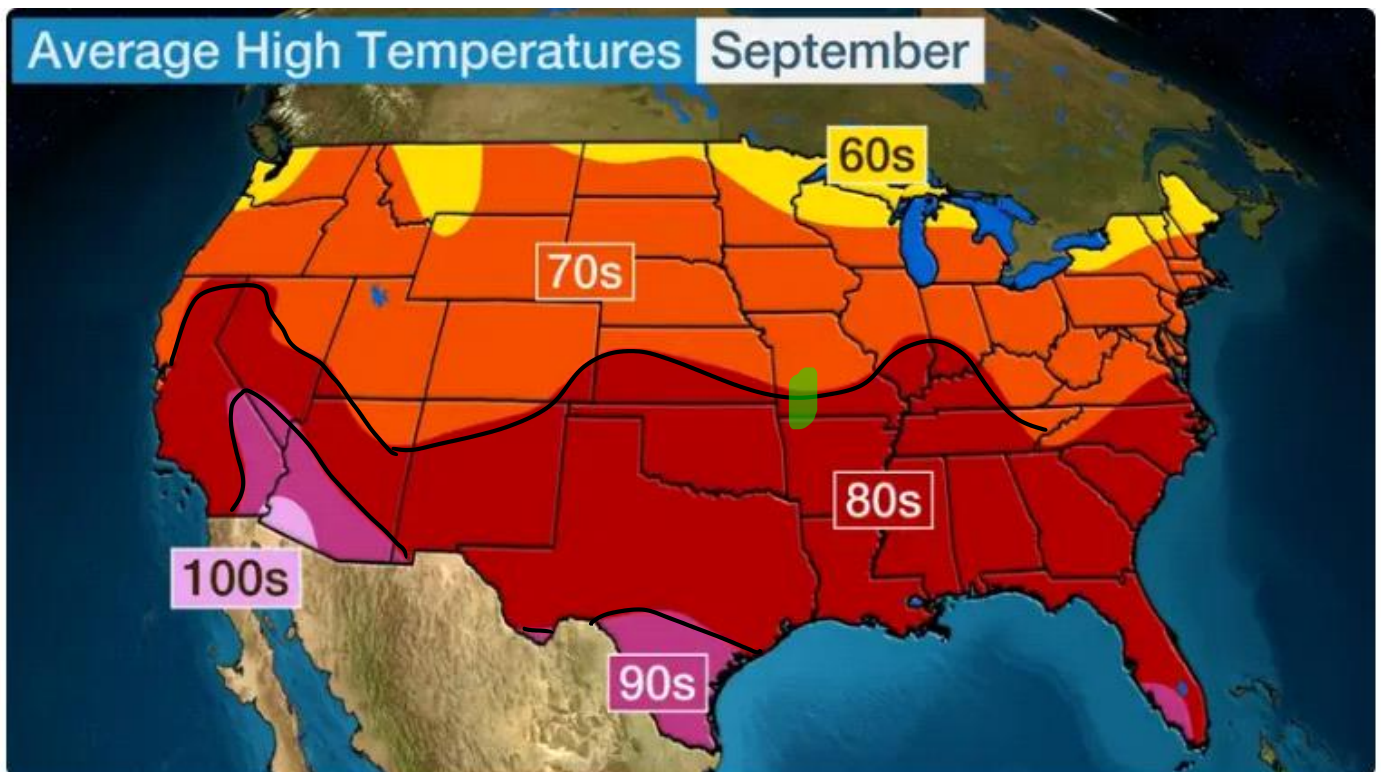
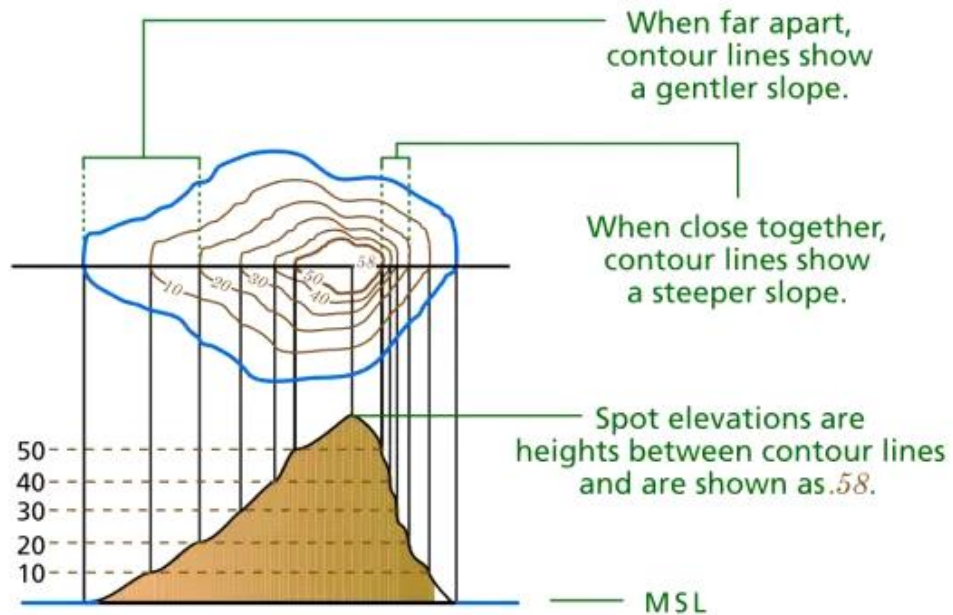
$$y - x^2 \geq 0$$

$$y \geq x^2$$

$$D = \left\{ (x, y) \mid x \neq \pm 1 \text{ and } y \geq x^2 \right\}$$



Definition: The level (contour) curves of a function of two variable are the curves with the equations $f(x, y) = k$, where k is a constant in the range of f .



Example: Sketch level curves for $f(x, y) = x^2 + y^2$ } paraboloid.

$$z = x^2 + y^2$$

pick z value

$$z=0$$

$$0 = x^2 + y^2$$

$$z=1$$

$$1 = x^2 + y^2$$

$$z=2$$

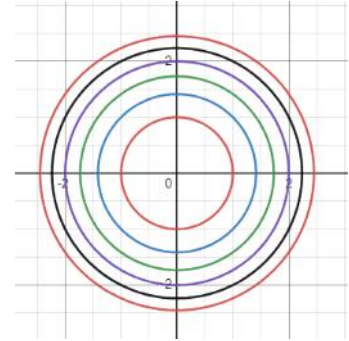
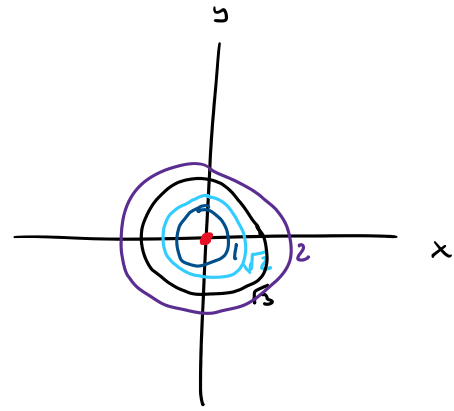
$$2 = x^2 + y^2$$

$$z=3$$

$$3 = x^2 + y^2$$

$$z=4$$

$$4 = x^2 + y^2$$



Example: Sketch a contour graph for $z = \ln(y^2 - x)$

Let $z=0$ $0 = \ln(y^2 - x)$

$$e^0 = y^2 - x$$

$$x = y^2 - e^0$$

$z=1$

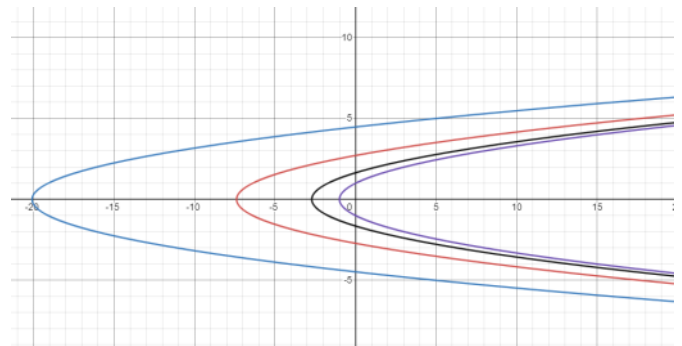
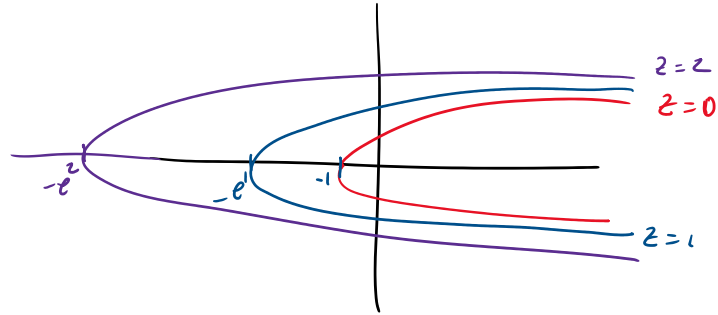
$$1 = \ln(y^2 - x)$$

$$e^1 = y^2 - x$$

$$x = y^2 - e^1$$

$z=2$ $x = y^2 - e^2$

domain is $y^2 - x > 0$
 $y^2 > x$



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Example: Find the domain of $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 25}}$

$$D = \left\{ (x, y, z) \mid x^2 + y^2 + z^2 > 25 \right\}$$

Example: Sketch the level surfaces for $f(x, y, z) = x^2 + y^2 - z$

$$w = x^2 + y^2 - z$$

Let $w = 0$

$$0 = x^2 + y^2 - z$$

$$z = x^2 + y^2$$

$w = 1$

$$1 = x^2 + y^2 - z$$

$$z = x^2 + y^2 - 1$$

$w = 2$ $z = x^2 + y^2 - 2$

