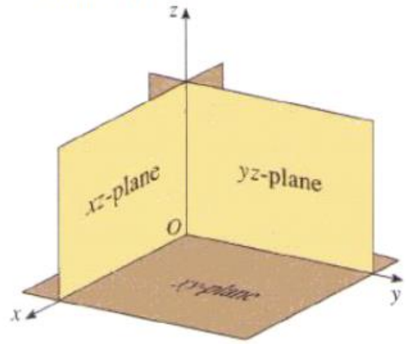
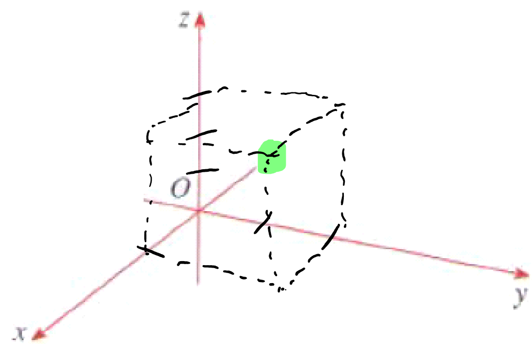
Section 12.1: Three Dimensional Coordinate System

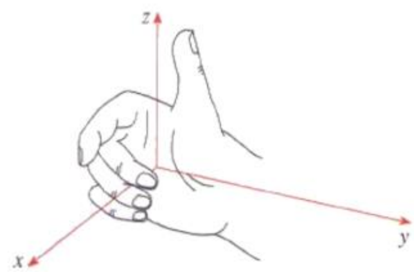
Coordinate planes



Coordinate axis



Right Hand Rule



x y z
(1, 2, 3)

Distance formula: The distance $|P_1P_2|$ between the points $P_1(x_1, y_1, z_1)$ and $P_2(x_2, y_2, z_2)$ is

$$|P_1P_2| = \sqrt{(x_2 - x_1)^2 + (y_2 - y_1)^2 + \underline{(z_2 - z_1)^2}}$$

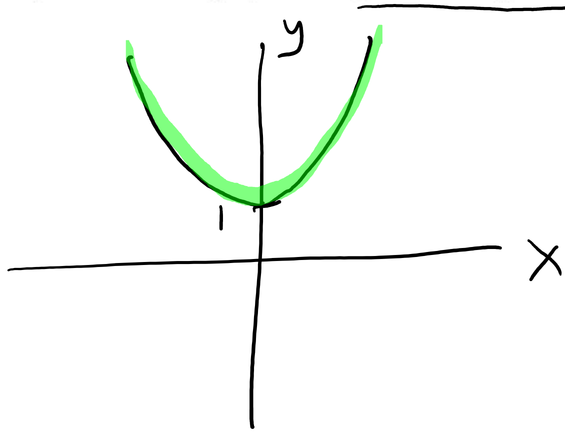
Shapes in 3-space

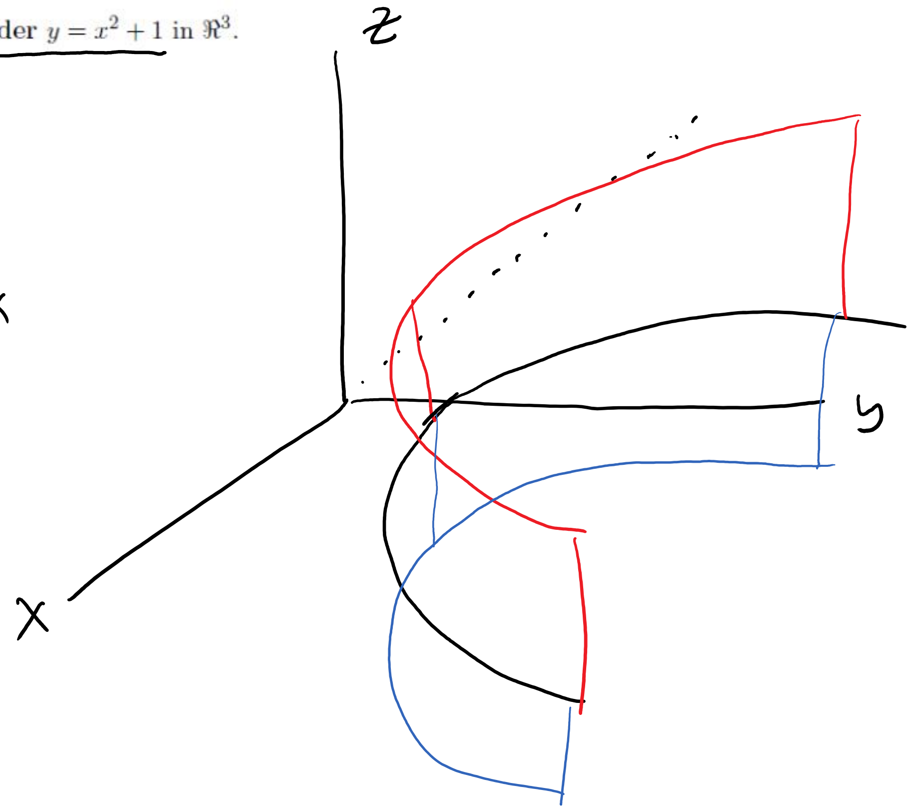
Cylindrical surfaces

An equation that only two of the variables x, y, and z represents a curve when graphed in $\Re^2$ and a cylindrical surface when graphed in $\Re^3$.

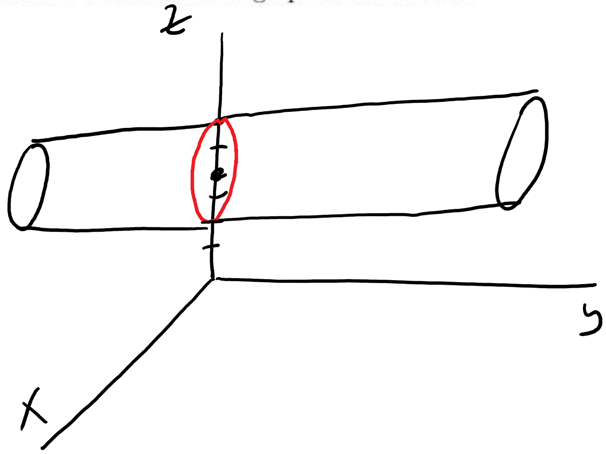
To graph the cylindrical surface, first graph the equation in the coordinate plane of the two variables and then translate that graph with respect to the axis of the missing variable.

Example: Sketch the graph of the **parabolic cylinder** $y = x^2 + 1$ in $\Re^3$.





Example: Let S be the graph of $x^2 + z^2 - 8z + 12 = 0$ in $\Re^3$
(a) Describe S and sketch a graph of the surface.



$x^2 + z^2 - 8z = -12$

$x^2 + z^2 - 8z + (4)^2 = -12 + (4)^2$

$x^2 + (z-4)^2 = 4$

center $(0,4)$ (x, z) $r = 2$

on the xz plane

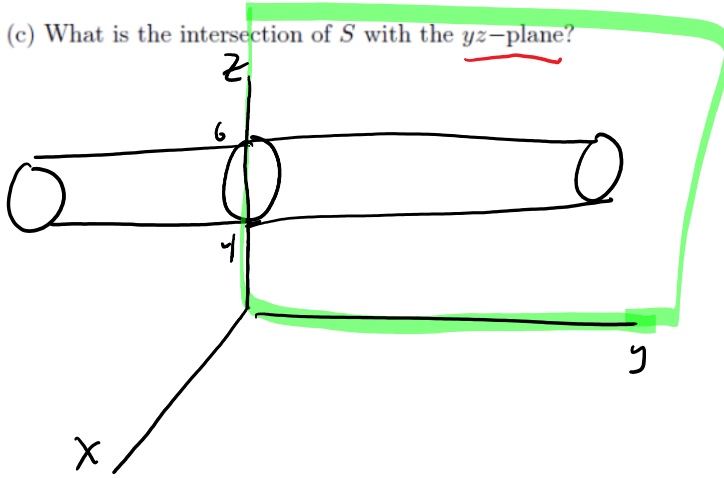
(b) What is the intersection of S with the xz−plane?

$y = 0$

circle $x^2 + (z-4)^2 = 4$

$$x^2 + (z-4)^2 = 4$$

(c) What is the intersection of S with the yz-plane?



$x = 0$

two horizontal lines

$$(z-4)^2 = 4$$
$$(z-4) = \pm 2$$
$$z = 4 \pm 2$$

$z = 6$	$z = 2$
$x = 0$	$x = 0$
$y =$ any #	$y =$ any #

(d) What is the intersection of S with the xy-plane?

none.

$z = 0$

$$x^2 + (0-4)^2 = 4$$
$$x^2 + 16 = 4$$
$$x^2 = -12$$ not possible

Plane

The equation of a plane is of the form $ax + by + cz = d$ with a, b, c, and d are constants.

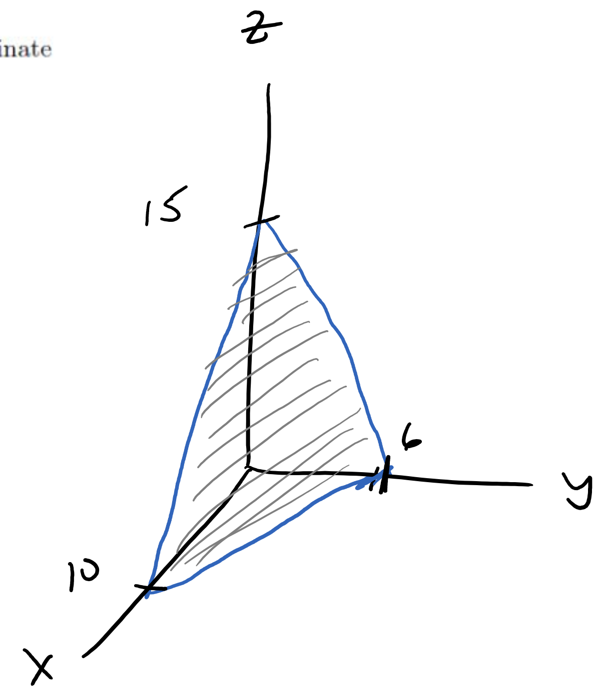
Example: Find the points where the plane $3x+5y+2z = 30$ intersects the coordinate axis. Sketch a graph of this plane.

x y z

(10 , 0 , 0)

(0 , 6 , 0)

(0 , 0 , 15)



Sphere An equation of a sphere with center $C(h,k,l)$ and radius r is

$$(x-h)^2+(y-k)^2+(z-l)^2=r^2$$

Example: Find an equation of a sphere with center at $(3,4,-1)$ and a radius of 7.

$$(x-3)^2+(y-4)^2+(z+1)^2=49$$

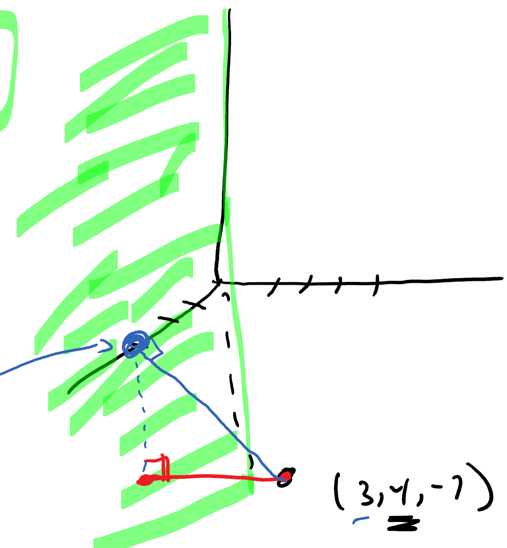
Example: Use the sphere $(x-3)^2 + (y-4)^2 + (z+7)^2 = 25$ to answer the following.
A) Find the intersection of the sphere and the xz coordinate plane.

$y = 0$

$$(x-3)^2 + (0-4)^2 + (z+7)^2 = 25$$

$$(x-3)^2 + 16 + (z+7)^2 = 25$$

$$\boxed{(x-3)^2 + (z+7)^2 = 9}$$



B) What is the distance from the center of the sphere to the xz-plane?

center $(3, 4, -7)$

distance $= 4$

$(3, 0, 0)$

C) How far is the center from the x-axis?

$$d = \sqrt{(3-3)^2 + (0-4)^2 + (0--7)^2}$$

$$= \sqrt{0 + 16 + 49}$$

$$= \sqrt{65}$$

Example: Find the center and radius of this sphere.

$2x^2 + 2y^2 + 2z^2 + 8y - 6z = 4$

$$x^2 + \underline{y^2 + 4y} + \underline{z^2 - 3z} = 2$$

$$x^2 + y^2 + 4y + (2)^2 + z^2 - 3z + \left(\frac{3}{2}\right)^2 = 2 + (2)^2 + \left(\frac{3}{2}\right)^2$$

$$x^2 + (y+2)^2 + \left(z - \frac{3}{2}\right)^2 = 8.25$$

center $\left(0, -2, \frac{3}{2}\right)$ $\quad r = \sqrt{8.25}$

Example: Describe the following region of $\Re^3$ represented by the equation(s)
$x^2 + z^2 = 10$, $y = 4$

in the plane $y=4$ circle

Radius is $r=\sqrt{10}$

cent. $(0, 4, 0)$