

Section 14.1: Functions of Several Variables

$$f(x) = x^2$$

Definition: A function f of two variables is a rule that assigns to each ordered pair of real numbers (x, y) in a set D a unique real number denoted by $z = f(x, y)$. The set D is the **domain** of f and its **range** is the set of values that f takes on, that is $\{f(x, y) | (x, y) \in D\}$.

Definition: If f is a function of two variables with domain D , the **graph** of f is the set $\{(x, y, z) \in \mathbb{R}^3 \mid z = f(x, y), (x, y) \in D\}$.

Example: Find the domain and sketch the graph of the function. What is the range?

(a) $f(x, y) = 10 - x - 2y$

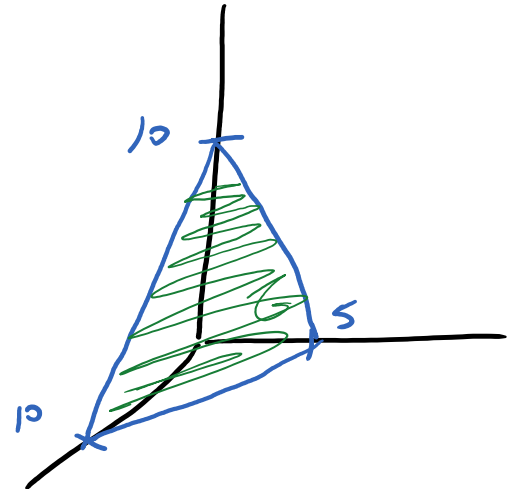
Domain is all reals for x
all reals for y

$$z = 10 - x - 2y$$

$$x + 2y + z = 10$$

$\mathbb{R} \times \mathbb{R} \rightarrow xy \text{ plane.}$
 $\searrow \mathbb{R}^2$

Range is all reals



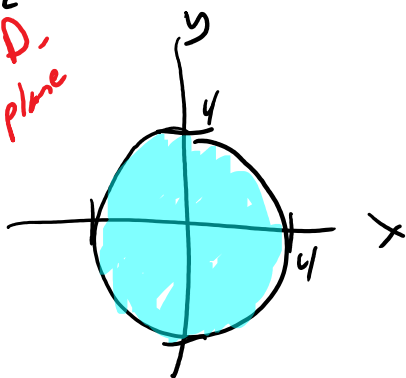
$$(b) f(x, y) = \sqrt{16 - x^2 - y^2}$$

$$16 - x^2 - y^2 \geq 0$$

$$16 \geq x^2 + y^2$$

$$D = \left\{ (x, y) \mid x^2 + y^2 \leq 16 \right\}$$

Graph of D
in the xy plane



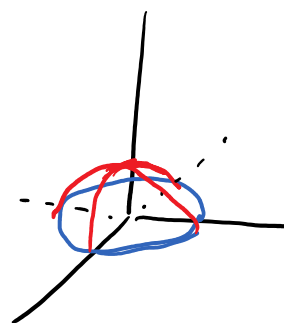
Range: $0 \leq f(x, y) \leq 4$

$$0 \leq z \leq 4$$

$$z = \sqrt{16 - x^2 - y^2}$$

$$z^2 = 16 - x^2 - y^2$$

$$\underline{x^2 + y^2 + z^2 = 16}$$



hemisphere.

Example: Find and sketch the domain.

(a) $z = \frac{\sqrt{y-x^2}}{1-x^2}$

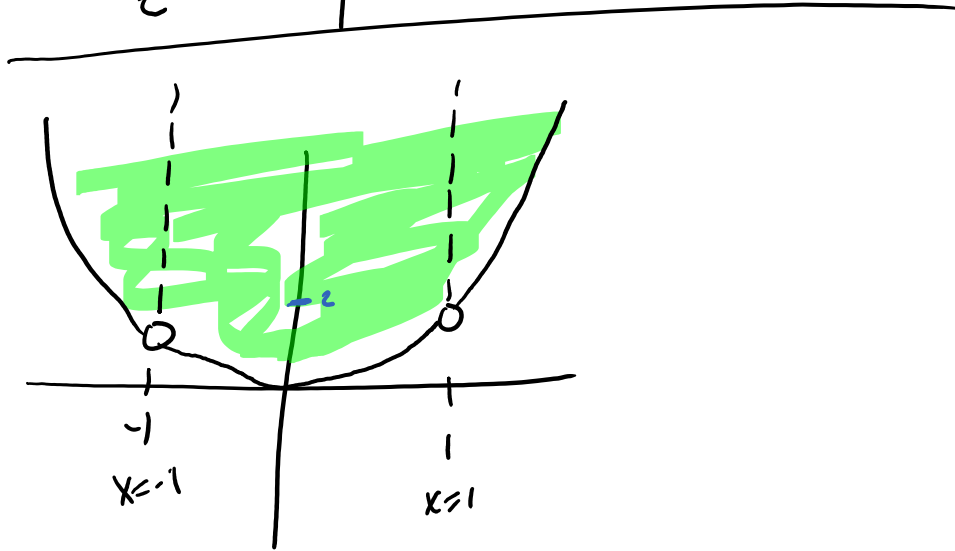
$x \neq \pm 1$

$y - x^2 \geq 0$

$y \geq x^2$

$z = f(x, y)$

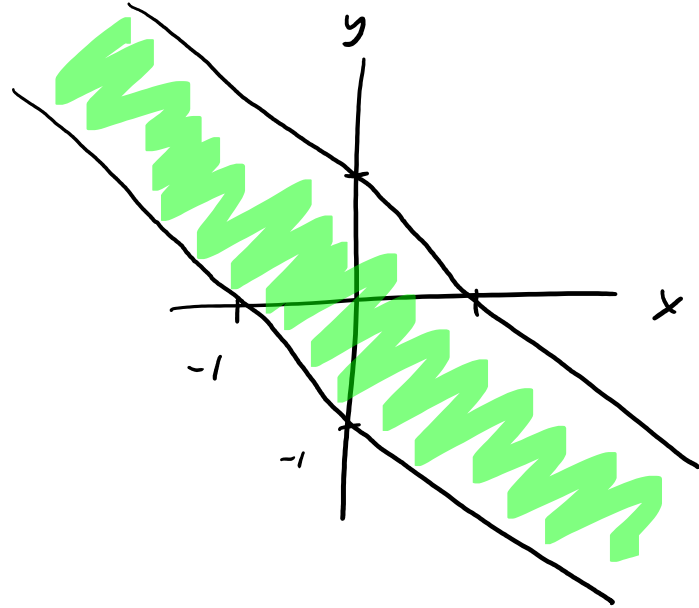
$D = \{(x, y) \mid y \geq x^2 \text{ and } x \neq \pm 1\}$



$$(b) f(x, y) = \sin^{-1}(x + y)$$

$$-1 \leq x + y \leq 1$$

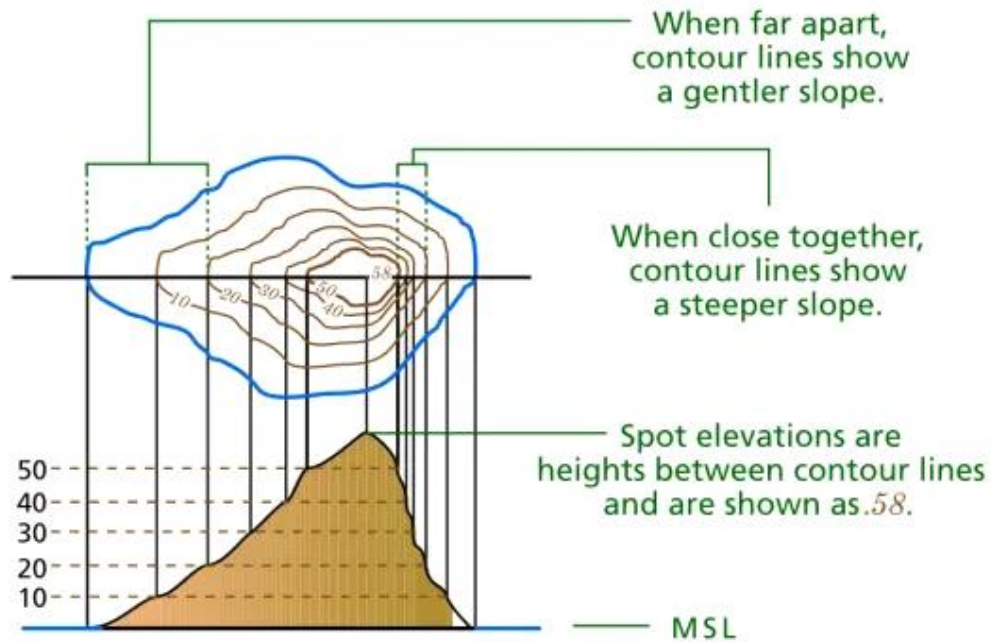
$$D = \left\{ (x, y) \mid -1 \leq x + y \leq 1 \right\}$$



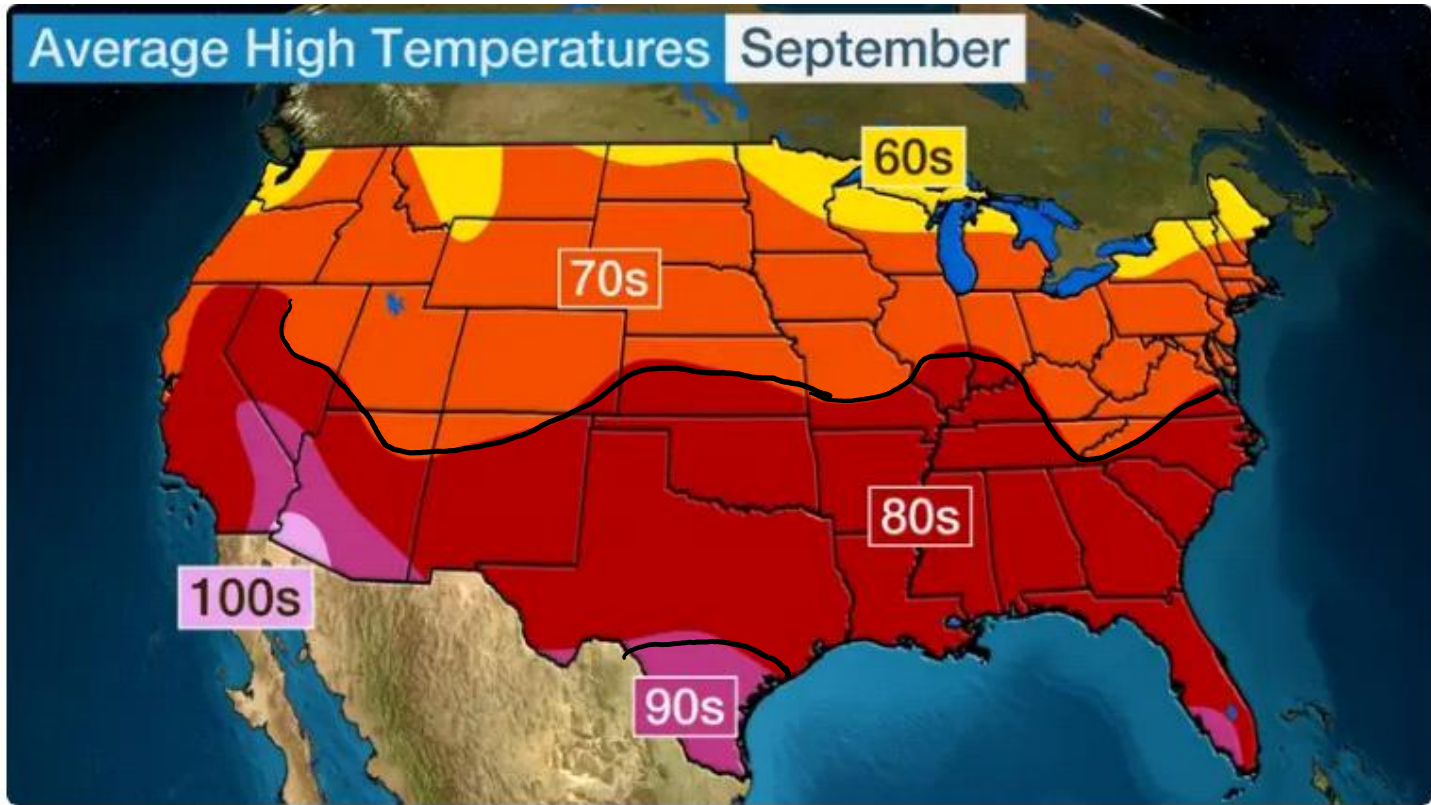
$$x + y = 1$$

$$x + y = -1$$

Definition: The level (contour) curves of a function of two variable are the curves with the equations $f(x, y) = k$, where k is a constant in the range of f .



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$$z = f(x, y)$$

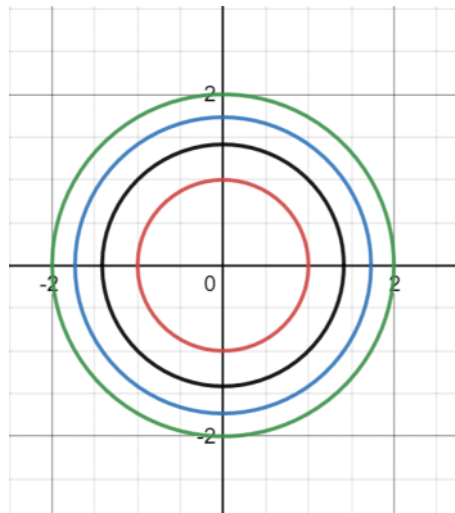
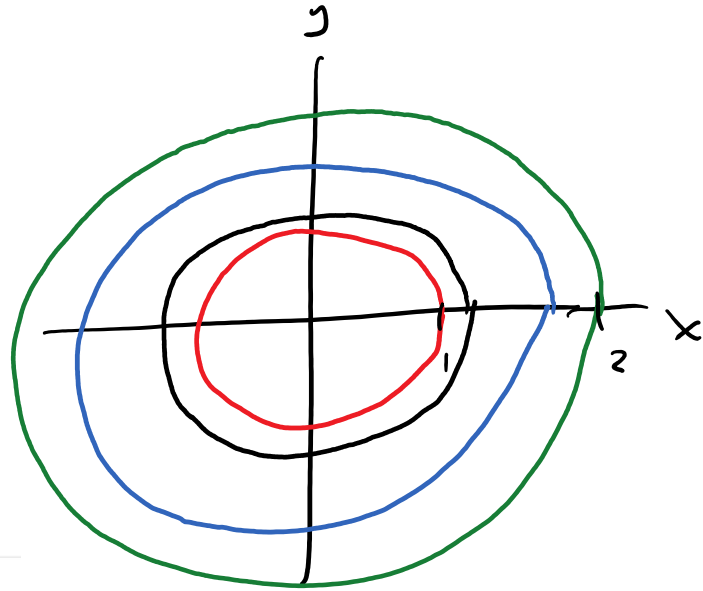
Example: Sketch level curves for $f(x, y) = x^2 + y^2$

$$z=1 \quad 1 = x^2 + y^2$$

$$z=2 \quad 2 = x^2 + y^2$$

$$z=3 \quad 3 = x^2 + y^2$$

$$z=4 \quad 4 = x^2 + y^2$$



Example: Sketch a contour graph for $z = \ln(y^2 - x)$

$$y^2 - x > 0$$

$$y^2 > x$$

$$x < y^2$$

$$z=1$$

$$1 = \ln(y^2 - x)$$

$$e^1 = y^2 - x$$

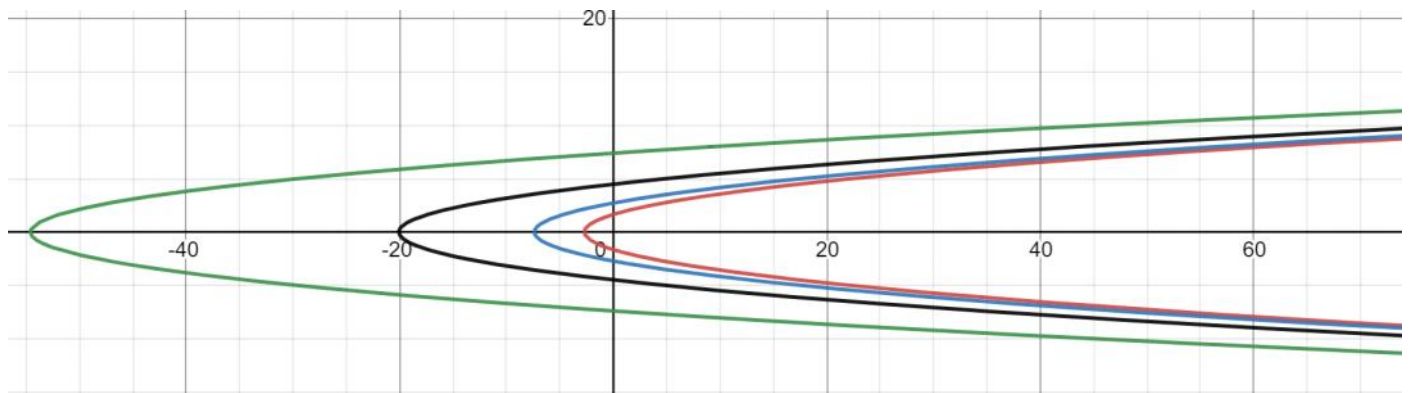
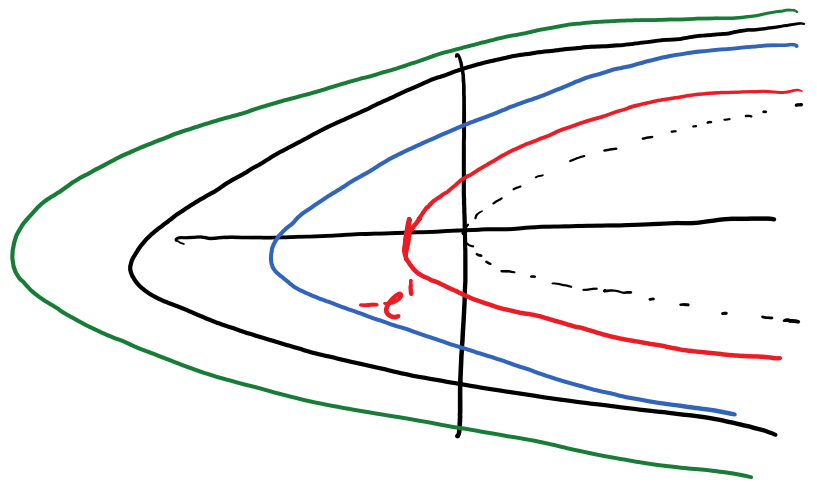
$$x = y^2 - e^1$$

$$z=2 \quad x = y^2 - e^2$$

$$z=3 \quad x = y^2 - e^3$$

$$z=4$$

$$x = y^2 - e^4$$



Example: Find the domain of $f(x, y, z) = \frac{1}{\sqrt{x^2 + y^2 + z^2 - 25}}$

$$x^2 + y^2 + z^2 - 25 > 0$$

$$D = \left\{ (x, y, z) \mid x^2 + y^2 + z^2 > 25 \right\}$$

Example: Sketch the level surfaces for $f(x, y, z) = x^2 + y^2 - z$

$$f(x, y, z) = 0$$

$$z = x^2 + y^2$$

$$f(x, y, z) = 1$$

$$\underline{z = x^2 + y^2 - 1}$$

$$f(x, y, z) = 2$$

$$\underline{z = x^2 + y^2 - 2}$$

