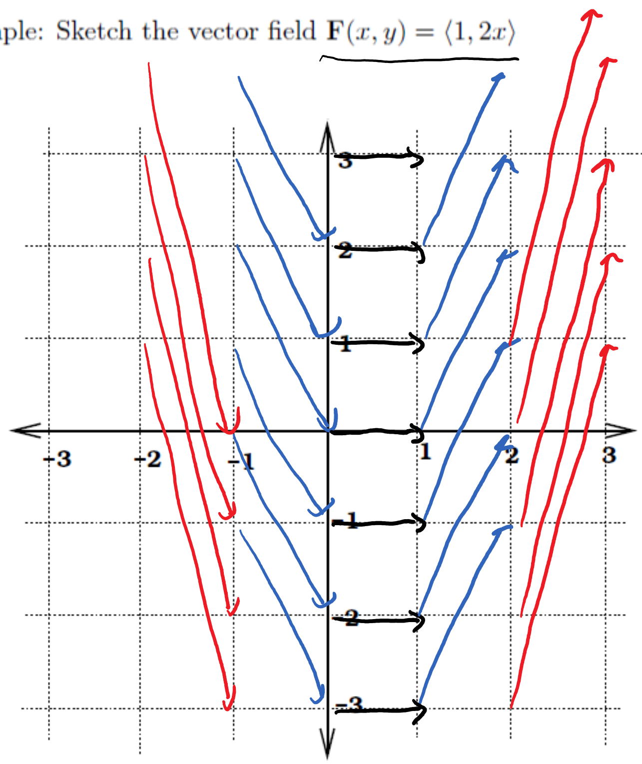
Section 16.1: Vector Fields

Definition: Let D be a set of points in either $\Re^2$ or $\Re^3$. A **vector field** is a function **F** that assigns to each point in D to a vector.

in $\Re^2$: $\mathbf{F}(x,y) = \langle P_1(x,y), P_2(x,y)\rangle$

in $\Re^3$: $\mathbf{F}(x,y,z) = \langle P_1(x,y,z), P_2(x,y,z), P_3(x,y,z)\rangle$

Example: Sketch the vector field $\mathbf{F}(x,y) = \langle 1, 2x\rangle$



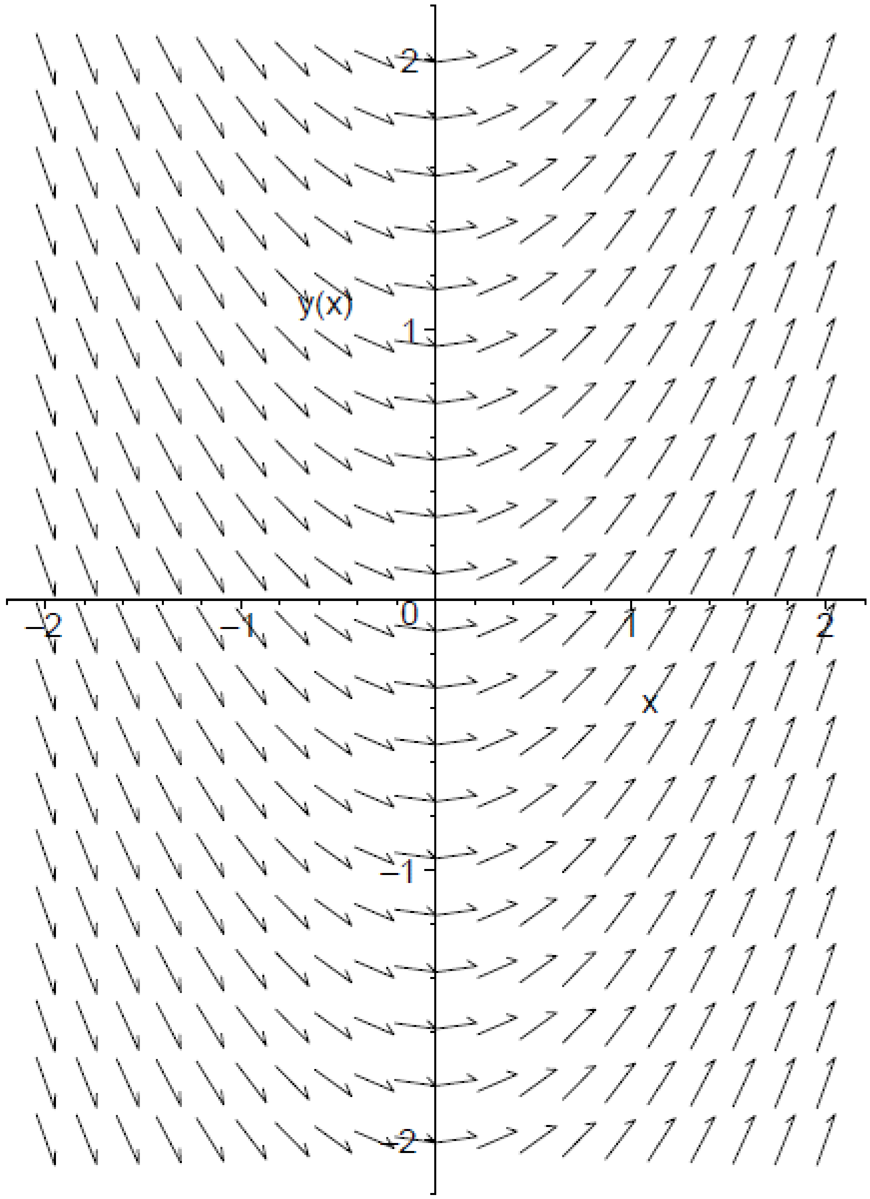
$F(0,*) = \langle 1,0\rangle$

$F(1,*) = \langle 1,2\rangle$

$F(-1,*) = \langle 1,-2\rangle$

$F(2,*) = \langle 1,4\rangle$

$F(-2,*) = \langle 1,-4\rangle$


y(x)
x
2
1
0
–1
–2
–2
–1
1
2

Definition: If f is a scalar function then the gradient of f, $\nabla f = \langle f_x, f_y \rangle$, is called a **gradient vector field**.

A vector field F is called a **conservative vector field** if it is the gradient of some scalar function f, i.e. $\mathbf{F}(x, y) = \nabla f(x, y)$. The function f is called a **potential function** for $\mathbf{F}$.

Example: Find the gradient vector field for $f(x, y, z) = x \ln(y - z)$.

$$\nabla f = \langle f_x, f_y, f_z \rangle$$

$$= \underbrace{\left\langle \ln(y-z), \frac{x}{y-z}, \frac{-x}{y-z} \right\rangle}$$