Section 4.2: Standard Minimization Problems

Minimization with $\leq$ constraints:

Example: Solve the linear programming problem.

minimize $C = -2x - 3y + 4z$

$-x + 2y - z \leq 8$

$x - 2y + 2z \leq 10$

$2x + 4y - 3z \leq 12$

$x, y, z \geq 0$

Standard Minimization Problems:
1) Objective function is minimized.
2) All variables are non-negative.
3) All constraints are in the form:
   $ax + by + \ldots \geq \text{constant}$

Dual Problems: Every standard minimization linear programming problem is associated with a standard maximization problem (and vice versa). The original problem is called the **primal problem** and the associated problem is called the **dual problem**.
The Fundamental Theorem of Duality:
A primal problem has a solution if and only if the corresponding dual problem has a solution. Furthermore, if a solution exists, then:

A) The objective function of both the primal and the dual problems attain the same optimal value.

B) The optimal solution to the problem appears under the slack variables in the simplex tableau associated with the dual problem.

Example: Find the dual problem and give the solution to the minimization (primal) problem.

Minimize $C = 40x + 12y + 40z$

$2x + y + 5z \geq 20$

$4x + y + z \geq 30$

$x, y, z \geq 0$
Example: The shipping problem from the section 3.2 notes.

Minimize \( C = 16x + 20y + 22z + 18u + 16v + 14w \)

\[
x + y + z \leq 800 \\
u + v + w \leq 600 \\
x + u \geq 500 \\
y + v \geq 400 \\
z + w \geq 450 \\
x, y, z, u, v, w \geq 0
\]

\[
P = \begin{bmatrix}
-1 & -1 & -1 & 0 & 0 & 0 & -800 \\
0 & 0 & 0 & -1 & -1 & -1 & -600 \\
1 & 0 & 0 & 1 & 0 & 0 & 500 \\
0 & 1 & 0 & 0 & 1 & 0 & 400 \\
0 & 0 & 1 & 0 & 0 & 1 & 450 \\
16 & 20 & 22 & 18 & 16 & 14 & 0
\end{bmatrix}
\]

\[
P^T = \begin{bmatrix}
-1 & 0 & 1 & 0 & 0 & 16 \\
-1 & 0 & 1 & 0 & 0 & 20 \\
-1 & 0 & 1 & 0 & 0 & 22 \\
0 & -1 & 1 & 0 & 0 & 18 \\
0 & -1 & 0 & 1 & 0 & 16 \\
0 & -1 & 0 & 0 & 1 & 14 \\
-800 & -600 & 500 & 400 & 450 & 0
\end{bmatrix}
\]

Note: 4 steps thru the simplex method: R1C3, R6C5, R5C4, R2C2.