Horizontal and Vertical Asymptote Shortcuts

The function \( f(x) \) has a **vertical asymptote** at \( x = a \) provided one of the following conditions are true.

\[
\lim_{x \to a^+} f(x) = \infty \quad \lim_{x \to a^-} f(x) = -\infty \quad \lim_{x \to a^-} f(x) = \infty \quad \lim_{x \to a^+} f(x) = -\infty
\]

A function \( f(x) \) has a **horizontal asymptote** of \( y = a \) provided

\[
\lim_{x \to \infty} f(x) = a \quad \text{or} \quad \lim_{x \to -\infty} f(x) = a
\]

If the function \( f(x) \) is a rational function, a polynomial divided by a polynomial, then we have some shortcuts for finding asymptotes.

**Shortcut for Vertical Asymptotes:** To find the VA of a rational function, first need to simplify the function. Factor the numerator and the denominator and cancel any common terms. The function will have VA where the simplified function is not defined.

**Shortcut for Horizontal Asymptotes:** If \( f(x) \) is a rational function of the form

\[
f(x) = \frac{a_n x^n + a_{n-1} x^{n-1} + \ldots + a_0}{b_m x^m + b_{m-1} x^{m-1} + \ldots + b_0}
\]

where \( x^n \) is the highest power in the numerator and \( x^m \) is the highest power in the denominator.

<table>
<thead>
<tr>
<th>Case</th>
<th>HA</th>
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</thead>
<tbody>
<tr>
<td>( n = m )</td>
<td>( y = \frac{a_n}{b_m} )</td>
</tr>
<tr>
<td>( n &lt; m )</td>
<td>( y = 0 )</td>
</tr>
<tr>
<td>( n &gt; m )</td>
<td>no HA</td>
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