MATH 251 Spring 2017
EXAM IV - VERSION A

LAST NAME: ______________________ FIRST NAME: ______________________

SECTION NUMBER: __________

UIN: ______________________

DIRECTIONS:

1. You may use a calculator on this exam. Programming formulas into the calculator is considered academic dishonesty.
2. TURN OFF cell phones and put them away. If a cell phone is seen during the exam, your exam will be collected and you will receive a zero.
3. In Part 1 (Problems 1-11), mark the correct choice on your ScanTron using a No. 2 pencil. The ScanTron will not be returned, therefore *for your own records, also record your choices on your exam!* Each problem is worth 4 points.
4. In Part 2 (Problems 12-16), present your solutions in the space provided. *Show all your work* neatly and concisely and *clearly indicate your final answer.* You will be graded not merely on the final answer, but also on the **quality** and **correctness** of the work leading up to it.
5. Be sure to *write your name, section number and version letter of the exam on the ScanTron form.*

THE AGGIE CODE OF HONOR

"An Aggie does not lie, cheat or steal, or tolerate those who do."

Signature: ______________________

DO NOT WRITE BELOW!

Question Type	Points Awarded	Points
Multiple Choice		55
Free Response		45
Total		100

PART I: Multiple Choice. 5 points each.

1. Evaluate $\int_C (\sin x + \cos y) ds$, where C is the line segment going from the point $(0, 0)$ to the point $(3\pi, 4\pi)$.

   (a) $\frac{2}{3\pi}$

   (b) $\frac{10}{3}$

   (c) $-\frac{10}{3}$

   (d) $30\pi^2$

   (e) $-\frac{2}{3\pi}$

2. Find the surface area of the part of the plane $z - x - y = 4$ that lies inside the cylinder $x^2 + y^2 = 9$.

   (a) $9\sqrt{2}\pi$

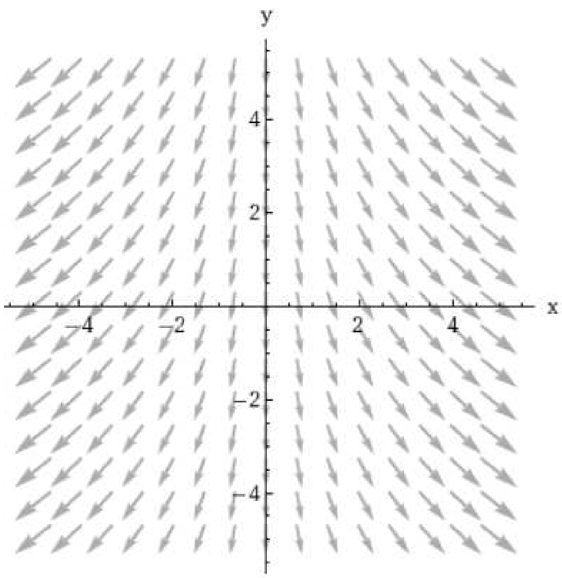
   (b) $6\sqrt{3}\pi$

   (c) $\sqrt{19}\pi$

   (d) $9\sqrt{3}\pi$

   (e) $2\sqrt{19}\pi$

3. Which of the following vector functions $\mathbf{F}$ has the vector field shown below?



(a) $\mathbf{F} = \langle -2x, -8 \rangle$

(b) $\mathbf{F} = \langle 2x, -8 \rangle$

(c) $\mathbf{F} = \langle 2x, 8 \rangle$

(d) $\mathbf{F} = \langle -2x, 8 \rangle$

(e) None of these

4. Using the Divergence Theorem, find the flux of $\mathbf{F} = \left\langle e^z + \cos y,\ y^3 + \sin(xz),\ z^3 + 2e^{-x} \right\rangle$ across the positively oriented surface S, where S is the part of the cylinder $y^2 + z^2 = 1$ that lies between the planes $x = 0$ and $x = 3$.

(a) $\dfrac{3\pi}{2}$

(b) 6π

(c) $\dfrac{9\pi}{2}$

(d) 3π

(e) $\dfrac{\pi}{2}$

5. Find the work done by the force field $\mathbf{F} = \langle x^2 + y^2,\ xy \rangle$ in moving a particle along the curve C: $\mathbf{r}(t) = \langle t^2, t^3 \rangle$, $0 \le t \le 1$.

   (a) $\frac{20}{21}$

   (b) $\frac{52}{21}$

   (c) $\frac{23}{24}$

   (d) $\frac{22}{56}$

   (e) $\frac{167}{68}$

6. Find $\oint_C y^2 dx + x dy$ where C is the triangle with vertices $(0,0)$, $(1,1)$ and $(0,1)$. Assume positive (counterclockwise) orientation.

   (a) $\frac{7}{6}$

   (b) $\frac{1}{6}$

   (c) $-\frac{7}{6}$

   (d) 0

   (e) $-\frac{1}{6}$

7. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle 2xy^3z^4,\ 3x^2y^2z^4,\ 4x^2y^3z^3 \rangle$ and C is any path from the point $(0,0,0)$ to the point $(1,1,1)$.

(a) 2

(b) 1

(c) -2

(d) 0

(e) -1

8. Evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle e^{x-1},\ xy \rangle$ and C: $\mathbf{r}(t) = \langle t^2,\ t^3 \rangle,\ \ 0 \le t \le 1$.

(a) $\frac{11}{22} - \frac{1}{2e}$

(b) $\frac{17}{22}$

(c) $\frac{11}{8}$

(d) $\frac{11}{8} - \frac{e}{2}$

(e) $\frac{11}{8} - \frac{1}{e}$

9. Which of the following integrals is the correct set up in order to evaluate $\iint_S xz\,dS$ where S is the part of the sphere $x^2+y^2+z^2=1$ that lies between the planes $z=0$ and $z=\dfrac{1}{2}$? Note: If we parameterize the sphere $x^2+y^2+z^2=\rho^2$ by $\mathbf{r}(\theta,\phi)=\langle \rho\sin(\phi)\cos(\theta),\ \rho\sin(\phi)\sin(\theta),\ \rho\cos(\phi)\rangle$, then $|\mathbf{r}_\theta\times\mathbf{r}_\phi|=\rho^2\sin(\phi)$.

(a) $\displaystyle\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\sin^2(\phi)\cos(\phi)\cos(\theta)\,d\phi d\theta$

(b) $\displaystyle\int_0^{2\pi}\int_{\pi/3}^{\pi/2}\sin^3(\phi)\cos(\phi)\cos(\theta)\,d\phi d\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^{\pi/3}\sin^3(\phi)\cos(\phi)\cos(\theta)\,d\phi d\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^{\pi/3}\sin^2(\phi)\cos^2(\phi)\,d\phi d\theta$

(e) $\displaystyle\int_0^{2\pi}\int_{\pi/6}^{\pi/2}\sin^2(\phi)\cos(\phi)\cos(\theta)\,d\phi d\theta$

10. Which of the following integrals is a result of using Stokes' Theorem to find $\displaystyle\int_C \mathbf{F}\cdot d\mathbf{r}$ where $\mathbf{F}=\langle -y^2,x,z^2\rangle$ and C is the curve of intersection of $z=9-x^2-y^2$ and the xy plane. Orient C to be counterclockwise when looking from above.

(a) $\displaystyle\int_0^{2\pi}\int_0^3(r+2r^2\sin\theta)\,drd\theta$

(b) $\displaystyle\int_0^{\pi}\int_0^3(1+2r\sin\theta)\,drd\theta$

(c) $\displaystyle\int_0^{2\pi}\int_0^3(r-2r^2\sin\theta)\,drd\theta$

(d) $\displaystyle\int_0^{2\pi}\int_0^3(1+2r\sin\theta)\,drd\theta$

(e) $\displaystyle\int_0^{\pi}\int_0^3(1-2r\sin\theta)\,drd\theta$

11. Given $\mathbf{F}=\langle x+y^2,\ 2xy+y^2\rangle$, find $f(-1,2)$ where f is the potential function of $\mathbf{F}$.

(a) $-\dfrac{2}{3}$

(b) $-\dfrac{5}{6}$

(c) $\dfrac{2}{3}$

(d) 2

(e) $\dfrac{23}{6}$

Part II: Work out. For problems (13-15), you are being asked for the set up only. Be sure your limits of integration are defined and matched with the appropriate differential.

12. (7 pts) Let f be a scalar field and let $\mathbf{F}$ be a vector field. Determine whether each expression is meaningful or meaningless (circle one). **If the expression is meaningful, circle whether the expression is a vector or a scalar**.

a.) curl $\mathbf{F}$	meaningful (vector or scalar)	meaningless
b.) grad$f$	meaningful (vector or scalar)	meaningless
c.) grad (div $\mathbf{F}$)	meaningful (vector or scalar)	meaningless
d.) (grad f) $\times$ (curl $\mathbf{F}$)	meaningful (vector or scalar)	meaningless
d.) curl f	meaningful (vector or scalar)	meaningless
e.) grad $\mathbf{F}$	meaningful (vector or scalar)	meaningless
f.) div (grad$f$)	meaningful (vector or scalar)	meaningless

13. (10 pts) Consider $\int_C (\sin(xy)dx + (x + \ln(y+1)dy$, where C is the triangle with vertices $(0,0)$, $(2,1)$ and $(3,0)$. Using Green's Theorem, set up the resulting double integral. Do not evaluate the integral.

14. (10 pts) Set up but do not evauate the surface integral obtained by using Stokes' Theorem to evaluate $\displaystyle\int_C \mathbf{F} \cdot d\mathbf{r}$, where $\mathbf{F} = \langle y^2,\ z^2,\ x^2 \rangle$ and where C is the curve of intersection of of the plane $x + z = 1$ and the cylinder $x^2 + y^2 = 4$, oriented counterclockwise when viewed from above.

15. (8 pts) Set up but do not evaluate $\displaystyle\iint_S xyz\, dS$ where S is the part of the paraboloid $z = x^2 + y^2$ between the planes $z = 1$ and $z = 4$.

16. (10 pts) Using the The Divergence Theorem, find the flux of $\mathbf{F} = \left\langle z \arctan(y^2),\ z^3 \ln(x^2+1),\ z \right\rangle$ where S is the surface enclosed by the paraboloid $x^2 + y^2 + z = 2$ and the plane $z = 1$. Simplify your answer.