Problem 1

A triangle is formed by these vertices: $A(5,1,5)$, $B(3,2,3)$, and $C(1,4,4)$. Determine if the triangle is an isosceles triangle, a right triangle, both or neither.

Distance from A to B $d(\overline{AB}) = \sqrt{(5-3)^2+(1-2)^2+(5-3)^2}$

$= \sqrt{2^2+(-1)^2+(2)^2}$

$= \sqrt{4+1+4} = \sqrt{9} = 3$

$d(\overline{AC}) = \sqrt{(5-1)^2+(1-4)^2+(5-4)^2}$

$= \sqrt{4^2+(-3)^2+1^2}$

$= \sqrt{16+9+1} = \sqrt{26}$

$d(\overline{BC}) = \sqrt{(3-1)^2+(2-4)^2+(3-4)^2}$

$= \sqrt{2^2+(-2)^2+(-1)^2}$

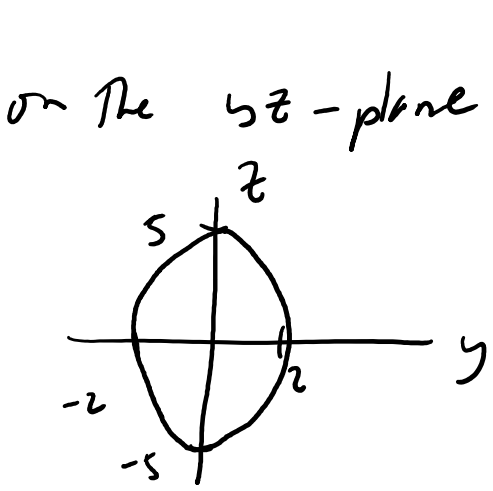
$= \sqrt{4+4+1} = \sqrt{9} = 3$

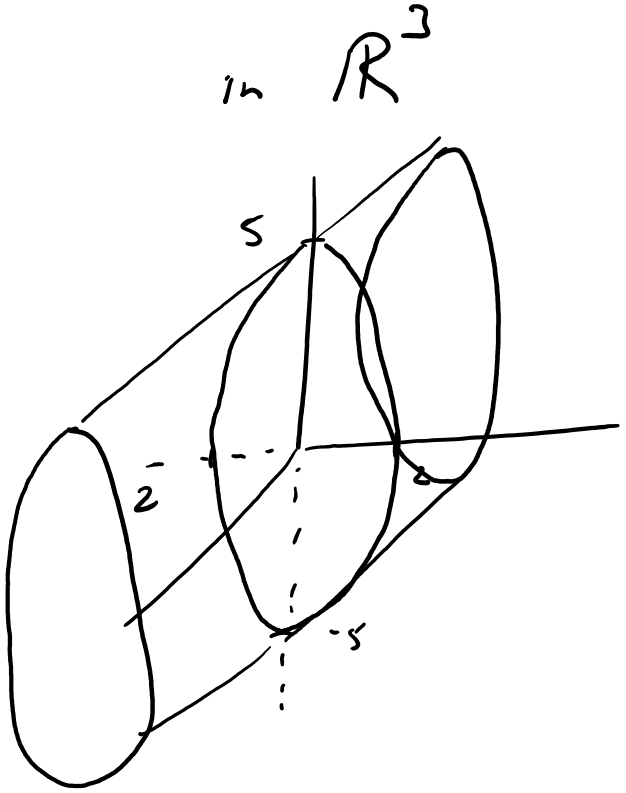
This is an isosceles triangle since 2 sides have the same length.

This is not a Right triangle since $3^2+3^2 = 18 \neq (\sqrt{26})^2$

Problem 2

Sketch the graph of the **elliptic cylinder** $\frac{y^2}{4} + \frac{z^2}{25} = 1$ in $\Re^3$.





Problem 3

Find an equation of a sphere that has a diameter with the endpoints of $(3, 2, 8)$ and $(7, 12, 22)$.

Length of the diameter $= \sqrt{(3-7)^2 + (2-12)^2 + (8-22)^2}$

$= \sqrt{(-4)^2 + (-10)^2 + (-14)^2}$

$= \sqrt{16 + 100 + 196}$

$= \sqrt{312} = 2\sqrt{78}$

radius : $\frac{1}{2}(2\sqrt{78}) = \sqrt{78}$

Center $= \left(\frac{3+7}{2}, \frac{2+12}{2} + \frac{8+22}{2}\right)$

$= (5, 7, 15)$

Answer : $(x-5)^2 + (y-7)^2 + (z-15)^2 = 78$

Problem 4

Find an equation of a plane that goes through the points $(10, 0, 0)$, $(0, 5, 0)$ and $(0, 0, 20)$

General form $ax + by + cz = d$

plugging in the points gives

$(10,0,0)$	$(0,5,0)$	$(0,0,20)$
$10a = d$	$5b = d$	$20c = d$

now pick a value for d & solve for a, b & c.

Let $d = 20$

$\Rightarrow$ $a = 2$ $b = 4$ and $c = 1$