Problem 1

Given $\mathbf{a} = \langle 2, -2, 1\rangle$ and $|\mathbf{b}| = 4$, what is the maximum value of $|\mathbf{a} \times \mathbf{b}|$.

$a \times b = (|a||b| \sin\theta)\, n$ $\quad$ θ is angle between $\vec{a}$ + $\vec{b}$

and n is a unit vector.

$|a \times b| = \big|\, |a|\,|b| \sin\theta \,\big|$ $\quad$ so max is when $\sin\theta = 1$ ie $\theta = \frac{\pi}{2}$

$|a| = \sqrt{4+4+1} = \sqrt{9} = 3$

max value is $3(4) = 12$ as long as the angle between $\vec{a}$ + $\vec{b}$ is $\frac{\pi}{2}$

Problem 2

Find a vector that is orthogonal to the plane $2x + 3y + 5z = 30$

In section 12.5 we will learn an easier method.

Lets find 2 vectors in the plane. ie need 3 points

That are all not in a line.

$A(0,0,6)$ $\quad\quad$ $\overrightarrow{AB} = \langle 0, 10, -6 \rangle$

$B(0,10,0)$ $\quad\quad$ $\overrightarrow{AC} = \langle 15, 0, -6 \rangle$

$C(15,0,0)$

$$\overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ 0 & 10 & -6 \\ 15 & 0 & -6 \end{vmatrix}$$

$$= \big(10(-6) - (0)(-6)\big)i - \big(0(-6) - (15)(-6)\big)j + \big((0)(0) - 15(10)\big)k$$

$$= \langle -60, -90, -150 \rangle$$

Any vector that is a scalar multiple of $\langle -60, -90, -150 \rangle$ will be a correct answer.

Problem 3

Find the volume of the parallelepiped determined by the vectors $\overset{A}{\langle 1,0,6\rangle}$, $\overset{B}{\langle 2,3,-8\rangle}$, and $\underset{C}{\langle 8,-5,6\rangle}$

$$A\cdot(B\times C) = \begin{vmatrix} 1 & 0 & 6 \\ 2 & 3 & -8 \\ 8 & -5 & 6 \end{vmatrix}$$

$$= 18 + 0 + (-60) - 144 - 40 - 0$$

$$= -226$$

$$\text{Volume} = |A\cdot(B\times C)| = 226$$

Problem 4

Are these vectors co-planer. Justify your answer.

a = 4i − 7j + k
b = −i + 4j + 2k
c = −i + 2j

$$a \cdot (b \times c) = \begin{vmatrix} 4 & -7 & 1 \\ -1 & 4 & 2 \\ -1 & 2 & 0 \end{vmatrix} = 0 + 14 + (-2) - (-4) - (16) - (0)$$

$$= 0$$

Since $a \cdot (b \times c) = 0$ These vector can all lie on the same plane. yes They are co-planer