Problem 1

If $f(t) = t^3$ and $\mathbf{r}(t) = \langle e^t, t^2 + 1, \sin(2t) \rangle$.

Compute $\dfrac{d}{dt} f(t)\mathbf{r}(t)$.

method 1 distribute the t^3 into the vector

$$\frac{d}{dt} t^3 r(t) = \frac{d}{dt} \langle t^3 e^t, t^5 + t^3, t^3 \sin(2t) \rangle$$

$$= \langle 3t^2 e^t + t^3 e^t, 5t^4 + 3t^2, 3t^2 \sin(2t) + t^3 \cdot 2\cos(2t) \rangle$$

method 2 Chain Rule

$$\frac{d}{dt} f(t) r(t) = f'(t) r(t) + f(t) \frac{d}{dt} r(t)$$

$$= 3t^2 \langle e^t, t^2 + 2, \sin(2t) \rangle + t^3 \langle e^t, 2t, 2\cos(2t) \rangle$$

Simplifying gives.

$$\langle 3t^2 e^t + t^3 e^t, 5t^4 + 3t^2, 3t^2 \sin(2t) + t^3 \cdot 2\cos(2t) \rangle$$

Problem 2

Let $\mathbf{r}(t) = \langle 2t, t^2, t^3 \rangle$.

(a) Find $T(t)$.

(b) Find an equation of the tangent line at $t = 3$.

(c) Find $\mathbf{r}'(t) \times \mathbf{r}''(t)$ at $t = 2$

A] $T(t)$ is the unit tangent vector function

$$T(t) = \frac{r'(t)}{r'(t)}$$

$$r' = \langle 2, 2t, 3t^2 \rangle \qquad |r'| = \sqrt{2^2 + (2t)^2 + (3t^2)^2} = \sqrt{4 + 4t^2 + 9t^4}$$

$$T(t) = \frac{1}{\sqrt{4+4t^2+9t^4}} \langle 2, 2t, 3t^2 \rangle$$

or

$$T(t) = \left\langle \frac{2}{\sqrt{4+4t^2+9t^4}}, \frac{2t}{\sqrt{4+4t^2+9t^4}}, \frac{3t^2}{\sqrt{4+4t^2+9t^4}} \right\rangle$$

B] for the tangent line at $t=3$ we need $r(3)$ and $r'(3)$

$$r(3) = \langle 6, 9, 27 \rangle \qquad r'(3) = \langle 2, 6, 27 \rangle$$

answer: $\boxed{\text{tangent line} = \langle 6+2t, 9+6t, 27+27t \rangle}$

C]

Problem 2 cont.

Let $\mathbf{r}(t) = \langle 2t, t^2, t^3 \rangle$.

(a) Find $T(t)$.

(b) Find an equation of the tangent line at $t = 3$.

(c) Find $\mathbf{r}'(t) \times \mathbf{r}''(t)$ at $t = 2$

method 1) find cross product and then evaluate for $t=2$

$r'(t) = \langle 2, 2t, 3t^2 \rangle$

$r''(t) = \langle 0, 2, 6t \rangle$

$$r'(t) \times r''(t) = \begin{vmatrix} i & j & k \\ 2 & 2t & 3t^2 \\ 0 & 2 & 6t \end{vmatrix}$$

$$= \left[2t(6t) - 2(3t^2)\right] i - \left[2(6t) - 0(3t^2)\right] j + \left[2(2) - (0)(2t)\right] k$$

$$= 6t^2 i - 12t j + 4k = \langle 6t^2, -12t, 4 \rangle$$

at $t=0$ we get $\langle 24, -24, 4 \rangle$

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method 2) find $r'(2)$ and $r''(2)$ and then do the cross product

$r'(2) = \langle 2, 4, 12 \rangle$ $\qquad$ $r''(2) = \langle 0, 2, 12 \rangle$

$$r'(2) \times r''(2) = \begin{vmatrix} i & j & k \\ 2 & 4 & 12 \\ 0 & 2 & 12 \end{vmatrix}$$

$$= \left[4(12) - 2(12)\right] i - \left[2(12) - 0(12)\right] j + \left[2(2) - 0(4)\right] k$$

$$= \left[4(12) - 2(12)\right] i - \left[2(12) \quad \ldots \right] j \ldots$$

$$= 24i - 24j + 4k = \langle 24, -24, 4 \rangle$$

Problem 3

Find a vector equation for the tangent line to the curve of intersection of the cylinders $x^2 + y^2 = 25$ and $y^2 + z^2 = 20$ at the point $(3, 4, 2)$.

First we need a space curve that represents the intersection of cylinders.

Since both cylinders have a y-variable let $y = t$

This gives $x^2 + t^2 = 25$ and $t^2 + z^2 = 20$

$x^2 = 25 - t^2$ $\qquad z^2 = 20 - t^2$

$x = \pm\sqrt{25 - t^2}$ $\qquad z = \pm\sqrt{20 - t^2}$

Since we were given the point $(3,4,2)$ we know $x > 0$ and $z > 0$ so

$r(t) = \left\langle \sqrt{25-t^2},\ t,\ \sqrt{20-t^2} \right\rangle$ and we want the tangent line at $t = 4$ $(y = 4)$

$$r'(t) = \left\langle \tfrac{1}{2}(25-t^2)^{-1/2}(-2t),\ 1,\ \tfrac{1}{2}(20-t^2)^{-1/2}(-2t) \right\rangle$$

$$r'(t) = \left\langle \frac{-t}{\sqrt{25-t^2}},\ 1,\ \frac{-t}{\sqrt{20-t^2}} \right\rangle$$

$$r'(4) = \left\langle \frac{-4}{\sqrt{25-16}},\ 1,\ \frac{-4}{\sqrt{20-16}} \right\rangle$$

$$= \left\langle \frac{-4}{\sqrt{9}}, 1, \frac{-4}{\sqrt{4}} \right\rangle = \left\langle -\frac{4}{3}, 1, \frac{-4}{2} \right\rangle$$

$$r'(4) = \left\langle -\frac{4}{3}, 1, -2 \right\rangle$$

now $r(4) = \langle 3, 4, 2 \rangle$

$$\boxed{\text{Tangent line} = \left\langle 3 - \frac{4}{3}t, 4+t, 2 - 2t \right\rangle}$$