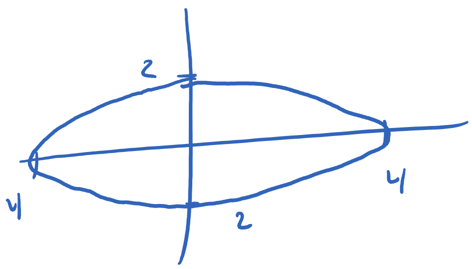
Problem 1

Find and sketch the domain: $f(x, y) = \ln(16 - x^2 - 4y^2)$

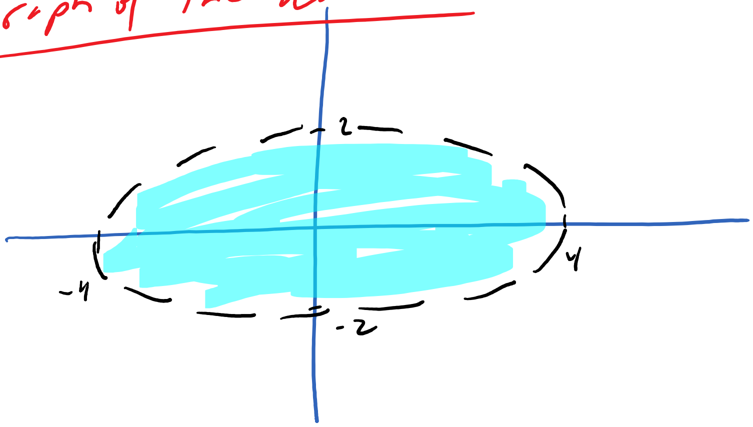
need $16 - x^2 - 4y^2 > 0$

or $16 > x^2 + 4y^2$

now $x^2 + 4y^2 = 16$ is an ellipse



graph of the domain



$D = \{ (x,y) \mid x^2 + 4y^2 < 16 \}$

Problem 2

Find and sketch the domain: $f(x, y) = \dfrac{\ln(2-x)}{1-x^2-y^2}$

for $\ln(2-x)$

need $2-x>0$

OR $2>x$ or $x<2$

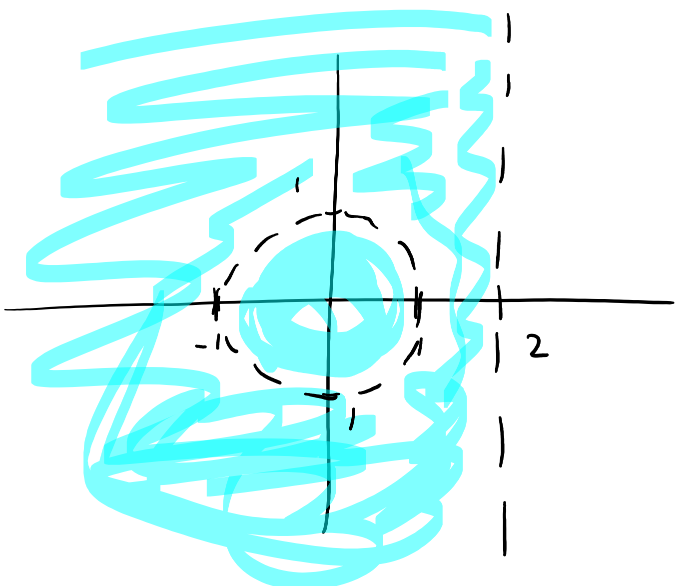
$D = \{(x,y) \mid x<2 \text{ and } x^2+y^2 \neq 1\}$

$1-x^2-y^2=0$

$\Rightarrow 1 = x^2+y^2$

is a circle centered at the origin with radius 1

So no point on the circle will work since this would be division by zero.



i.e. everything to the left of $x=2$ and excludes the circle $x^2+y^2=1$

Problem 3

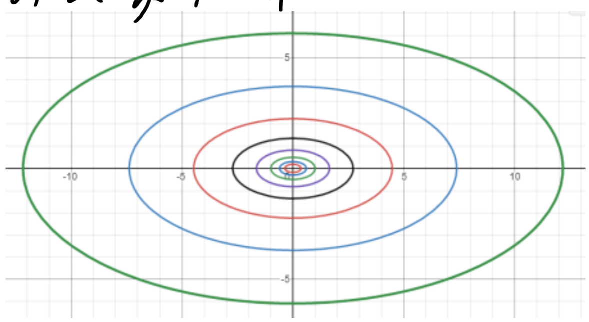
Sketch level curves(traces) for this function. What are the shapes of these level curves? Find two points that are on the graph of the level curve $f(x, y) = 3$
$f(x, y) = \ln(x^2 + 4y^2)$

The level curves are when $f(x,y) = k$ (some constant)

$$K = \ln(x^2 + 4y^2)$$
$$e^K = x^2 + 4y^2$$

This formula shows that for any value of k you get an ellipse.

Actual graph of the level curves.



Set $K = 3$ we get

$$e^3 = x^2 + 4y^2$$

we only need 2 points so let $y = 0$

gives $x^2 = e^3$

Thus $x = \pm\sqrt{e^3} = \pm e^{3/2}$
$= \pm e^{1.5}$

$$\boxed{\begin{aligned} x &= e^{1.5} \\ y &= 0 \\ f(x,y) &= 3 \end{aligned}}$$

$$\boxed{\begin{aligned} x &= -e^{1.5} \\ y &= 0 \\ f(x,y) &= 3 \end{aligned}}$$

Problem 4

Sketch level curves(traces) for this function. What are the shapes of these level curves? Find two points that are on the graph of the level curve $f(x, y) = 3$

$f(x, y) = \sqrt[3]{x^2 + y^2}$

$$K = \sqrt[3]{x^2+y^2}$$

$$K^3 = x^2+y^2$$

Level curves are circles.

Let $K=3 \longrightarrow 27 = x^2+y^2$

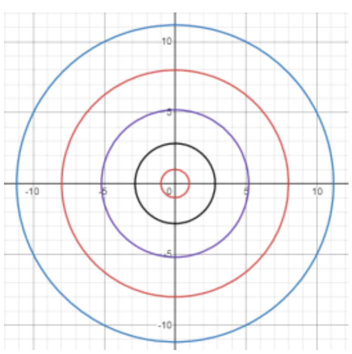
for the 2 points pick a value for x or y.

Let $y=1$ so $27 = x^2+(1)^2$

$26 = x^2$

$x = \pm\sqrt{26}$

actual graph of the level curves



points

$x=\sqrt{26}$ $y=1$ $f(x,y)=3$	$x=-\sqrt{26}$ $y=1$ $f(x,y)=3$

Problem 5

Determine the shape of the level surfaces for $f(x, y, z) = 10 + x^2 + 3y^2 + 4z^2$

level surfaces are when $f(x,y,z) = K$

$$K = 10 + x^2 + 3y^2 + 4z^2$$

or $$K - 10 = x^2 + 3y^2 + 4z^2$$

if $K < 10$ no shape.

$K = 10$ just the point at the origin.

$K > 10$ shapes are ellipsoids.