Problem 1

$f(x,y) = x^3y^4 + e^{y^2-1}$

(a) Find the equation of the tangent plane to $f(x,y)$ at the point $(2,1,9)$.

(b) Find the equation of the normal line at the point $(2,1,9)$.

A normal vector for the tangent plane will be $n = \langle f_x(2,1), f_y(2,1), -1 \rangle$

$f_x = 3x^2y^4$

$f_y = 4x^3y^3 + 2ye^{y^2-1}$

$f_x(2,1) = 3(2)^2(1)^4 = 12$

$f_y(2,1) = 4(2)^3(1)^3 + 2(1)e^0$
$= 32 + 2$
$= 34$

$n = \langle 12, 34, -1 \rangle$

Tangent plane: $12(x-2) + 34(y-1) - (z-9) = 0$

normal line:

$x = 2 + 12t$
$y = 1 + 34t$
$z = 9 - t$

Problem 2

$f(x,y) = x^3y^4 + e^{y^2-1}$

(a) Find the linearization function at the point $(2,1)$.

(b) Use the linearization function at the point $(2,1)$ to approximate $f(1.9, 1.2)$

$f(2,1) = 2^3 \cdot 1^4 + e^{1-1}$
$= 8 + 1 = 9$

$f_x = 3x^2y^4$
$f_x(2,1) = 3(2)^2 \cdot 1^4$
$= 12$

$f_y = 4x^3y^3 + 2ye^{y^2-1}$
$f_y(2,1) = 4 \cdot (2)^3 (1)^3 + 2e^0$
$= 32 + 2$
$= 34$

$L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(y-1)$

$\boxed{L(x,y) = 9 + 12(x-2) + 34(y-1)}$

b) $L(1.9, 1.2) = 9 + 12(1.9-2) + 34(1.2-1)$
$= 9 + 12(-.1) + 34(.2)$
$= 9 - 1.2 + 6.8$
$= 9 + 5.6$
$= 14.6$

Problem 3

Find the differential of these function (total differential).

(a) $z = e^{-2x}\cos(2y)$

(b) $R = \alpha\beta^2\ln(\gamma)$

A) $dz = z_x \, dx + z_y \, dy$

$$dz = -2e^{-2x}\cos(2y)\,dx + (-2)e^{-2x}\sin(2y)\,dy$$

B) $dR = R_\alpha \, d\alpha + R_\beta \, d\beta + R_\gamma \, d\gamma$

$$dR = \beta^2\ln(\gamma)\,d\alpha + 2\alpha\beta\ln(\gamma)\,d\beta + \frac{\alpha\beta^2}{\gamma}\,d\gamma$$

Problem 4

The radius and the height of a right circular cylinder are measured as 3 in. and 8 in., respectively. The possible error of the radius is 0.05 in and a possible error in the height of 0.15 in. Use differentials to estimate the maximum error in the calculated volume of the cylinder.

$dr = .05$ $r = 3$

$V = \pi r^2 h$ $dh = .15$ $h = 8$

$dV = \pi 2rh\,dr + \pi r^2\,dh$

$= 2\pi(3)(8)(.05) + \pi(3)^2(.15)$

$dV = 3.75\pi$

$dV = 11.78097$