Problem 1

Find and classify the critical values of $f(x, y) = 2x^3 + xy^2 + 5x^2 + y^2$

$f_x = 6x^2 + y^2 + 10x$

$f_y = 2xy + 2y$

need $f_y = 0$

$0 = 2xy + 2y$

$0 = 2y(x+1)$

so $y = 0$ <u>OR</u> $x = -1$

$\boxed{y=0}$

$f_x = 0$

$0 = 6x^2 + 0 + 10x$

$0 = 6x^2 + 10x$

$0 = x(6x + 10)$

$x = 0 \qquad x = \frac{-10}{6} = \frac{-5}{3}$

points $(0, 0) \quad \left(-\frac{5}{3}, 0\right)$

$\boxed{x=-1}$

$f_x = 0$

$0 = 6(-1)^2 + y^2 + 10(-1)$

$0 = 6 + y^2 - 10$

$0 = y^2 - 4$

$y = \pm 2$

points $(-1, 2)$
$(-1, -2)$

$f_{xx} = 12x + 10$

$f_{yy} = 2x + 2$

$f_{xy} = 2y$

points	$K = f_{xx} f_{yy} - (f_{xy})^2$	f_{xx}	conclusion
$(0,0)$	$10(2) - (0)^2 > 0$	pos.	local min
$\left(-\frac{5}{3}, 0\right)$	$(-10)\left(-\frac{4}{3}\right) - (0)^2 > 0$	neg.	local max
[illegible]	$(-2)(0) - (4)^2 < 0$	—	saddle point

$\left(\frac{-5}{3}, 0\right)$	$(-10)(\ 3\ )$		
$(-1, 2)$	$(-2)(0) - (4)^2 < 0$	$-$	saddle point
$(-1, -2)$	$(-2)(0) - (-4)^2 < 0$	$-$	saddle point

Problem 2

Find the absolute maximum/absolute minimum for $f(x, y) = \sqrt{1 - x^2 - y^2}$.

Think about the shape.

$f(x,y) = z = \sqrt{1-x^2-y^2}$

or $z^2 = 1 - x^2 - y^2$

or $x^2 + y^2 + z^2 = 1$ This is a sphere of radius 1

So $f(x,y)$ is the upper portion of the sphere.

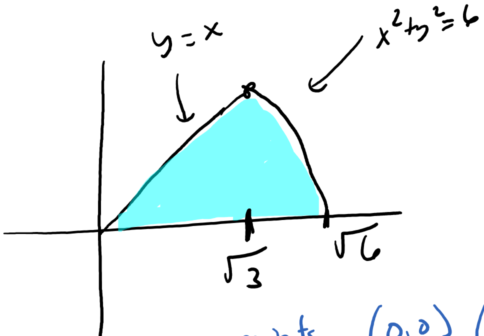
$\boxed{\begin{array}{l} \text{Abs max} = 1 \\ \text{Abs min} = 0 \end{array}}$

Problem 3

Find the absolute maximum/absolute minimum for $f(x, y) = xy^2 + 3$ on the set D.

$D = \left\{(x, y) \middle| \; x \geq 0, \;\; 0 \leq y \leq x, \;\; x^2 + y^2 \leq 6\right\}$

Step 1 figure out Region D.



Intersection of $y = x$ and the circle

$$x^2 + (x)^2 = 6$$
$$2x^2 = 6$$
$$x^2 = 3$$
$$x = \pm\sqrt{3}$$

note: corner points $(0,0)$ $(\sqrt{6},0)$ and $(\sqrt{3}, \sqrt{3})$ need to be Tested for Abs max/min.

Step 2) find any Regular critical values.

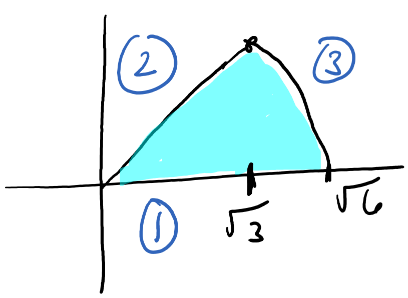
$$f(x,y) = xy^2 + 3$$

$f_x = y^2$ $\qquad$ $f_y = 2xy$

$f_x = 0$ when $y = 0$ $\qquad$ $f_y = 0$ when $x = 0$ or $y = 0$

Thus any point of the form $(x, 0)$ is a critical point. The entire x-axis.

step 3 find critical points for each of the sides.



Side 1 $\qquad y = 0 \qquad 0 \leq x \leq \sqrt{6}$

$$f(x, 0) = x(0)^2 + 3$$
$$f(x, 0) = 3$$

This side is constant so there are no critical points all function values for side 1 are 3.

Side 2

$y = x \qquad 0 \le x \le \sqrt{3}$

$$g = f(x, x) = x \cdot x^2 + 3$$
$$g = x^3 + 3$$
$$g' = 3x^2$$
$$g' = 0 \text{ when } x = 0$$

The only critical point is $(0, 0) \rightarrow \begin{matrix} x=0 \\ y=x=0 \end{matrix}$

Side 3

$y^2 = 6 - x^2$ or $y = +\sqrt{6-x^2}$ since y is positive and $\sqrt{3} \le x \le \sqrt{6}$

$$g = f(x, \sqrt{6-x^2}) = x\left(\sqrt{6-x^2}\right)^2 + 3$$
$$= x(6-x^2) + 3$$
$$g = 6x - x^3 + 3$$

$$g' = 6 - 3x^2$$

Set $g' = 0$ and solve

$$0 = 6 - 3x^2$$
$$3x^2 = 6$$
$$x^2 = 2$$
$$x = \sqrt{2}$$

Since $x = \sqrt{2}$ is not in the interval This side does not have any critical points.

Step 4 Test corner points and critical points.

points	$f(x,y) = xy^2 + 3$
$(0,0)$	3

$\boxed{\text{Abs max} = 3\sqrt{3} + 3}$

$(0,0)$	3
$(\sqrt{6},0)$	3
$(\sqrt{3},\sqrt{3})$	$3\sqrt{3}+3$
x-axis	3

$$\boxed{\begin{array}{l} \text{Abs max} = 3\sqrt{3}+3 \\ \text{Abs min} = 3 \end{array}}$$

Problem 4

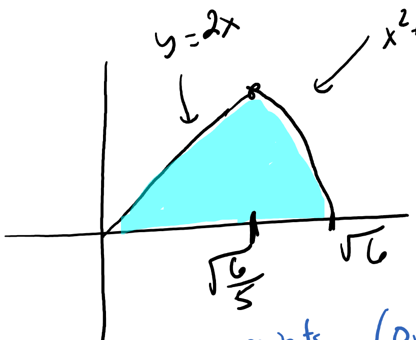
Find the absolute maximum/absolute minimum for $f(x,y) = xy^2 - x$ on the set D.

$$D = \left\{(x,y)\,\middle|\; x \geq 0,\; 0 \leq y \leq 2x,\; x^2 + y^2 \leq 6\right\}$$

Note: This problem is challenging. you need really need a calculator to do the evaluations

This is not a question that I would expect to see on ~~an~~ exam due to the numbers

Step 1 figure out Region D.



Intersection of $y=2x$ and the circle

$$x^2 + (2x)^2 = 6$$
$$5x^2 = 6$$
$$x^2 = 6/5$$
$$x = \sqrt{6/5}$$

note: corner points $(0,0)$ $(\sqrt{6},0)$ and $\left(\sqrt{\frac{6}{5}}, 2\sqrt{\frac{6}{5}}\right)$ need to be Tested for Abs max/min.

Step 2) find any Regular critical values.

$$f(x,y) = xy^2 - x$$

$f_x = y^2 - 1$ $\qquad f_y = 2xy$

$f_x = 0$ when $y = \pm 1$ $\qquad f_y = 0$ when $x=0$ or $y=0$

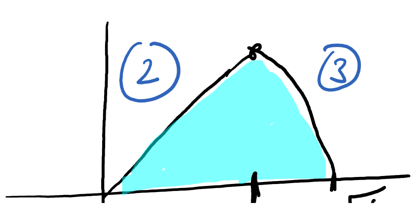
$y=1$ means $x=0$

$y=-1$ means $x=0$

Both of these points are not in the Region

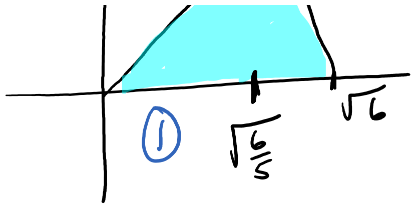
note $y=0$ is not a critical point since if $y=0$ then $f_x \neq 0$.

step 3 find critical points for each of the sides.



Side 1 $\quad y=0 \quad 0 \leq x \leq \sqrt{\frac{6}{5}}$

$$f(x,0) = x(0)^2 - x$$



$f(x, 0) = x(0)^2 - x$

$g = f(x, 0) = -x$

$g' = -1$ so no critical points for side 1

Side 2 $\quad y = 2x \quad 0 \le x \le \sqrt{\frac{6}{5}}$

$g = f(x, 2x) = x(2x)^2 - x$

$g = 4x^3 - x$

$g' = 12x^2 - 1$

$g' = 0 \quad 12x^2 - 1 = 0$

$12x^2 = 1$

$x^2 = \frac{1}{12}$

$x = \sqrt{\frac{1}{12}}$

critical point.

$x = \sqrt{\frac{1}{12}} = \frac{1}{\sqrt{12}}$

$y = \frac{2}{\sqrt{12}}$

Side 3 $\quad y^2 = 6 - x^2$ or $y = +\sqrt{6 - x^2}$ since y is positive and $\sqrt{\frac{6}{5}} \le x \le \sqrt{6}$

$\approx 1.095 \le x \le 2.45$

$g = f\left(x, \sqrt{6 - x^2}\right) = x\left(\sqrt{6 - x^2}\right)^2 - x$

$= x(6 - x^2) - x$

$g = 6x - x^3 - x = 5x - x^3$

$g' = 5 - 3x^2$

Set $g' = 0$ and solve

critical point.

$x = \sqrt{\frac{5}{3}}$

$y = \sqrt{6 - \left(\sqrt{\frac{5}{3}}\right)^2} = \sqrt{6 - \frac{5}{3}}$

Set $y = 0$ ...

$$0 = 5 - 3x^2$$
$$3x^2 = 5$$
$$x^2 = 5/3$$
$$x = \sqrt{5/3} \approx 1.24$$

$$y = \sqrt{6 - \left(\sqrt{\tfrac{5}{3}}\right)^2} = \sqrt{6 - \tfrac{5}{3}}$$
$$y = \sqrt{\tfrac{13}{3}}$$

Step 4 Test corner points and critical points.

points	$f(x,y) = xy^2 - x$
$(0,0)$	0
$(\sqrt{6},0)$	$-\sqrt{6} \approx -2.4495$
$\left(\sqrt{\frac{6}{5}}, 2\sqrt{\frac{6}{5}}\right)$	≈ 4.1627
Side 2 $\left(\frac{1}{\sqrt{12}}, \frac{2}{\sqrt{12}}\right)$	$\approx -.19245$
Side 3 $\left(\sqrt{\frac{5}{3}}, \sqrt{\frac{13}{3}}\right)$	≈ 4.3033

Abs max $= 4.3033$

Abs min $= -2.4495$