Problem 1

Use the method of Lagrange multipliers to find the point on the plane so that the functions $f(x,y,z)$ has the least value.

$f(x,y,z) = 4x^2 + y^2 + 5z^2$ $\qquad$ $2x + 3y + 4z = 12$

$g(x,y,z) = 2x + 3y + 4z$

$2x+3y+4z=12$

$f_x = \lambda g_x$ $\qquad$ $f_y = \lambda g_y$ $\qquad$ $f_z = \lambda g_z$

$8x = 2\lambda$ $\qquad$ $2y = 3\lambda$ $\qquad$ $10z = 4\lambda$

$4x = \lambda$ $\qquad$ $\frac{2}{3}y = \lambda$ $\qquad$ $\frac{5}{2}z = \lambda$

$4x = \lambda = \frac{2}{3}y$ $\qquad$ $4x = \lambda = \frac{5}{2}z$

$4x = \frac{2}{3}y$ $\qquad$ $4x = \frac{5}{2}z$

$6x = y$ $\qquad$ $\frac{8x}{5} = z$

Thus

$$2x + 3(6x) + 4\left(\frac{8x}{5}\right) = 12$$

$$2x + 18x + \frac{32x}{5} = 12$$

$$20x + \frac{32x}{5} = 12$$

$$\frac{132x}{5} = 12$$

$$x = \frac{60}{132} = \frac{30}{66} = \frac{5}{11}$$

So $y = 6x = \frac{30}{11}$ $\qquad$ $z = \frac{8x}{5} = \frac{8}{11}$

$\left(\frac{5}{11}, \frac{30}{11}, \frac{8}{11}\right)$ will be the poi[illegible]

with a min value.

Problem 2

Use the method of Lagrange multipliers to find the point on the plane so that the function $f(x, y, z)$ has the maximum value. assume that x, y, and $z \geq 0$.

$f(x, y, z) = xyz$ $\quad\quad$ $5x + y + 10z = 30$ $\quad\quad$ $g(x,y,z) = 5x + y + 10z$

$f_x = \lambda g_x$ $\quad\quad$ $f_y = \lambda g_y$ $\quad\quad$ $f_z = \lambda g_z$ $\quad\quad$ $5x + y + 10z = 30$

$yz = 5\lambda$ $\quad\quad$ $xz = \lambda$ $\quad\quad$ $xy = 10\lambda$

$\frac{yz}{5} = \lambda$ $\quad\quad$ $xz = \lambda$ $\quad\quad$ $\frac{xy}{10} = \lambda$

$\frac{yz}{5} = xz$

$yz = 5xz$

$yz - 5xz = 0$

$z(y - 5x) = 0$

$\underbrace{z = 0}_{}$ or $y = 5x$

not possible.

$\frac{yz}{5} = \frac{xy}{10}$

$10yz = 5xy$

$10yz - 5xy = 0$

$5y(2z - x) = 0$

$\underbrace{y = 0}_{}$ or $2z = x$

not possible

$z = \frac{x}{2}$

$5x + y + 10z = 30$

$5x + 5x + 5x = 30$

$15x = 30$

$x = 2$

$x = 2$ $\quad\quad$ $y = 10$

$z = 1$

point that max. $f(x,y,z)$

Problem 3

Use the method of Lagrange multipliers to find the point on the ellipsoid so that the function $f(x, y, z)$ has the maximum value.

$f(x, y, z) = 4x + 24y - 10z$ $\qquad$ $x^2 + 4y^2 + 5z^2 = 9$

$f_x = \lambda g_x$ $\qquad$ $f_y = \lambda g_y$ $\qquad$ $f_z = \lambda g_z$ $\qquad$ $x^2 + 4y^2 + 5z^2 = 9$

$4 = 2\lambda x$ $\qquad$ $24 = 8\lambda y$ $\qquad$ $-10 = 10\lambda z$

$2 = \lambda x$ $\qquad$ $3 = \lambda y$ $\qquad$ $-1 = \lambda z$

$2y = \lambda x y = 3x$ $\qquad$ $2z = \lambda z x = -x$

$2y = 3x$ $\qquad$ $2z = -x$

$y = \frac{3}{2}x$ $\qquad$ $z = -\frac{1}{2}x$

$x^2 + 4y^2 + 5z^2 = 9$

$x^2 + 4 \cdot \frac{9}{4}x^2 + 5 \cdot \frac{1}{4}x^2 = 9$

$x^2 + 9x^2 + \frac{5}{4}x^2 = 9$

$10x^2 + \frac{5}{4}x^2 = 9$

$\frac{45}{4}x^2 = 9$

$x^2 = 9 \cdot \frac{4}{45} = \frac{4}{5}$

$x = \pm \frac{2}{\sqrt{5}}$

$x = \frac{2}{\sqrt{5}}$

$y = \frac{3}{\sqrt{5}}$

$z = \frac{-1}{\sqrt{5}}$

This point is the max when evaluated in $f(x, y, z)$

It is positive.

$x = \frac{-2}{\sqrt{5}}$

$y = \frac{-3}{\sqrt{5}}$

$z = \frac{1}{\sqrt{5}}$