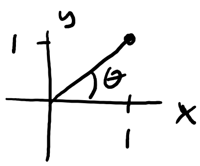
Problem 1

Convert these Cartesian points to cylindrical.

(a) $(1, 1, -1)$

(b) $(-1, -1, 3)$

A) $r = \sqrt{x^2 + y^2} = \sqrt{1^2 + 1^2} = \sqrt{2}$



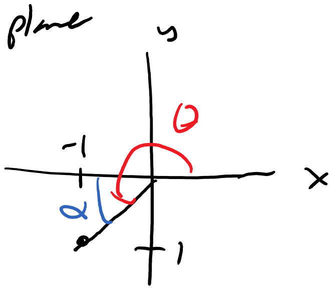
$\tan\theta = \frac{1}{1} = 1$

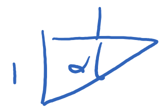
$\theta = \frac{\pi}{4}$

r, θ, z

$\left(\sqrt{2}, \frac{\pi}{4}, -1\right)$

B) xy plane





$\tan\alpha = 1$

$\alpha = \frac{\pi}{4}$

$\theta = \pi + \frac{\pi}{4}$

$\theta = \frac{3\pi}{4}$

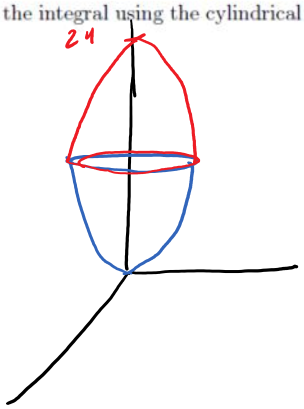
$r = \sqrt{x^2 + y^2}$

$r = \sqrt{(-1)^2 + (-1)^2} = \sqrt{2}$

$$\begin{array}{ccc} x & y & z \\ (-1, & -1, & 3) \end{array} \rightarrow \begin{array}{ccc} r & \theta & z \\ (\sqrt{2}, & \frac{3\pi}{4}, & 3) \end{array}$$

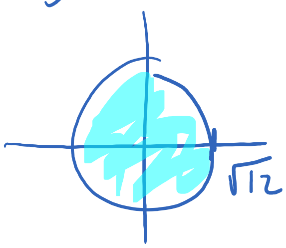
Problem 2

The solid E is the region bounded by the paraboloids $z = x^2+y^2$ and $z = 24-x^2-y^2$. Set up the integral using the cylindrical coordinate system to evaluate $\iiint_E x^2 z \, dV$



The projection on the xy plane is a circle. The radius of this circle will match the radius of Intersection of the two paraboloids.

Region D



$$x^2+y^2 = 24-x^2-y^2$$

$$2x^2+2y^2 = 24$$

$$x^2+y^2 = 12$$

Top $z = 24-x^2-y^2$ OR $z = 24-r^2$

Bottom $z = x^2+y^2$ OR $z = r^2$

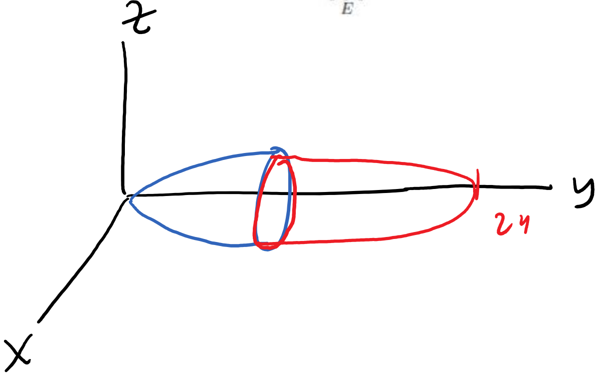
$0 \le r \le \sqrt{12}$

$0 \le \theta \le 2\pi$

$$\iiint_E x^2 z \, dV = \int_{\theta=0}^{2\pi} \int_{r=0}^{\sqrt{12}} \int_{z=r^2}^{24-r^2} r^2 \cos^2\theta \, z \cdot r \, dz\,dr\,d\theta$$

Problem 3

The solid E is the region bounded by the paraboloids $y = x^2+z^2$ and $y = 24-x^2-z^2$.
Set up the integral to evaluate $\iiint_E z^2y\,dV$



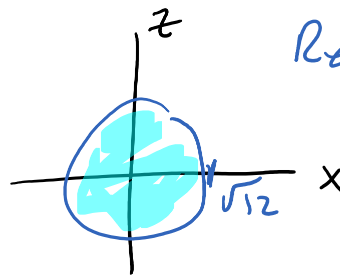
$x^2+y^2 = 24-x^2-y^2$

$2x^2+2y^2 = 24$

$x^2+y^2 = 12$

Left: $y = x^2+z^2$ or $y = r^2$

Right: $y = 24-x^2-y^2$ or $y = 24-r^2$



Region D

polar in xz is

$x = r\cos\theta$

$z = r\sin\theta$

$x^2+z^2 = r^2$

$$\iiint_E z^2y\,dv = \int_{\theta=0}^{2\pi}\int_{x=0}^{\sqrt{12}}\int_{y=r^2}^{24-r^2} r^2\sin^2\theta\, y \cdot r\,dy\,dr\,d\theta$$

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note you could convert this to a standard cylindrical method if you swap <u>every</u> y variable for z and <u>every</u> z variable for y.

Problem 4

Set up the integral, in cylindrical, to find the volume of the solid that is above the lower half of the sphere of radius 2, centered at the origin and below $z = 8-2(x^2+y^2)$ and in the half-space $y \geq 0$.

top : $z = 8-2(x^2+y^2)$

bottom : $z = -\sqrt{4-x^2-y^2}$

The intersection of the sphere $x^2+y^2+z^2=4$ and the paraboloid $z = 8-2(x^2+y^2)$ is needed

$$2(x^2+y^2) = 8-z$$
$$x^2+y^2 = \frac{1}{2}(8-z)$$

$$x^2+y^2+z^2=4$$
$$\frac{1}{2}(8-z)+z^2=4$$
$$8-z+2z^2=8$$
$$-z+2z^2=0$$
$$z(2z-1)=0$$
$$z=0 \quad z=\frac{1}{2}$$

$z=\frac{1}{2}$ is part of the upper part of the sphere so

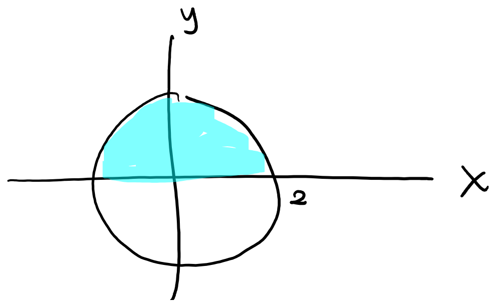
The answer is $z=0$

Intersection is $x^2+y^2+0^2=4$

or $x^2+y^2=4$

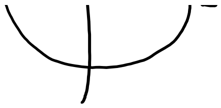
Region D

also we know $y \geq 0$



$$0 \leq r \leq 2$$
$$0 \leq \theta \leq \pi$$

Top : $z = 8-2(x^2+y^2) = 8-2r^2$



Top: $z = 8 - 2(x^2+y^2) = 8 - 2r^2$

Bottom: $z = -\sqrt{4-x^2-y^2} = -\sqrt{4-r^2}$

$$V = \iiint_E 1\,dV = \int_{\theta=0}^{\pi} \int_{r=0}^{2} \int_{z=-\sqrt{4-r^2}}^{8-2r^2} r\,dz\,dr\,d\theta$$

Problem 5

Convert the integral $\int_0^3 \int_0^{\sqrt{9-x^2}} \int_{\sqrt{3x^2+3y^2}}^{\sqrt{36-x^2-y^2}} z\sqrt{x^2+y^2+z^2}\; dzdydx$ to cylindrical

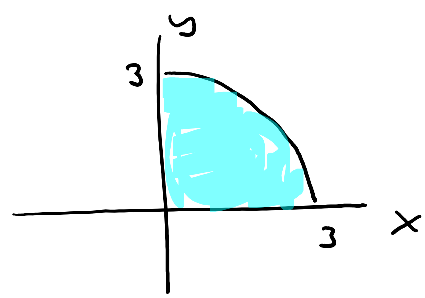
Top $z = \sqrt{36-x^2-y^2} = \sqrt{36-r^2}$

bottom $z = \sqrt{3x^2+3y^2} = \sqrt{3r^2} = \sqrt{3}\, r$

Region D

$0 \le y \le \sqrt{9-x^2}$

$0 \le x \le 3$



$0 \le \theta \le \pi/2$

$0 \le r \le 3$

$$\int_{\theta=0}^{\pi/2} \int_{r=0}^{3} \int_{z=\sqrt{3}\,r}^{\sqrt{36-r^2}} z\sqrt{r^2+z^2} \cdot r \; dz\, dr\, d\theta$$