Math 251 Exam 3 B Thursday, April 4, 2024

Printed Name: Key

Section: ______

UIN: ______

Signature: ______ On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

- **Show all appropriate work to receive full credit.**
- **You will be graded not merely on your final answer, but also on the quality and correctness of the work leading up to it.**
- If you need more space to work a problem, you may use the back of the cover page or the back of the exam. Please indicate where the problem is located.
- **Calculators are not allowed.**
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Good Luck!

1. (15 points) True or False. Circle your answer.

T F $\displaystyle\int_0^2 \int_{-\sqrt{4-x^2}}^{\sqrt{4-x^2}} g(x,y)dydx = \int_0^2 \int_0^{\pi} g(r\cos(\theta), r\sin(\theta))rd\theta dr$

T F If a, b, c, and d are constants such that $0 < a < b < c < d$, then

$$\int_{z=0}^{d} \int_{x=a}^{z+5} \int_{y=b}^{c} (2xy^2 + y^2z^2)\ dydxdz = \int_{y=b}^{c} y^2\ dy \int_{z=0}^{d} \int_{x=a}^{z+5} (2x + z^2)\ dxdz$$

T F The graph (in polar coordinates) of the equation $r = 9\cos(\theta)$ is a circle of radius 9.

T F The graph (in spherical coordinates) of the equation $\rho\cos(\theta)\sin(\phi) = 3$ is a plane.

T F Region D is described by the part of the xy−plane such that $a \le r \le b$, $c \le \theta \le d$.

$\displaystyle\int_{\theta=c}^{d} \int_{r=a}^{b} r\ drd\theta$ is equivalent to the area of region D.

2. (6 points) Give the equation in spherical coordinates. Simplify your answer.

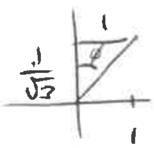
$$z = -\sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$$ $\leftarrow$ lower part of a cone

Consider $z = \sqrt{\frac{x^2}{3} + \frac{y^2}{3}}$ 

Answer $\phi = \pi - \frac{\pi}{3} = \frac{2\pi}{3}$

Let $x = 0$

$z = \frac{y}{\sqrt{3}}$ Let $y = 1$



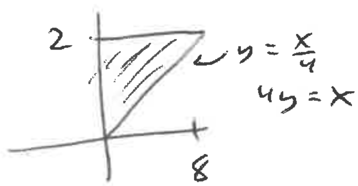
$\tan\phi = \frac{1}{\frac{1}{\sqrt{3}}} = \sqrt{3}$

$\phi = 60° = \frac{\pi}{3}$

3. (7 points) Evaluate the integral $\int_0^{x=8} \int_{y=x/4}^{2} \frac{\cos(y)}{y}\, dydx$ by reversing the order of integration.

$\frac{x}{4} \le y \le 2$

$0 \le x \le 8$



$0 \le y \le 2$

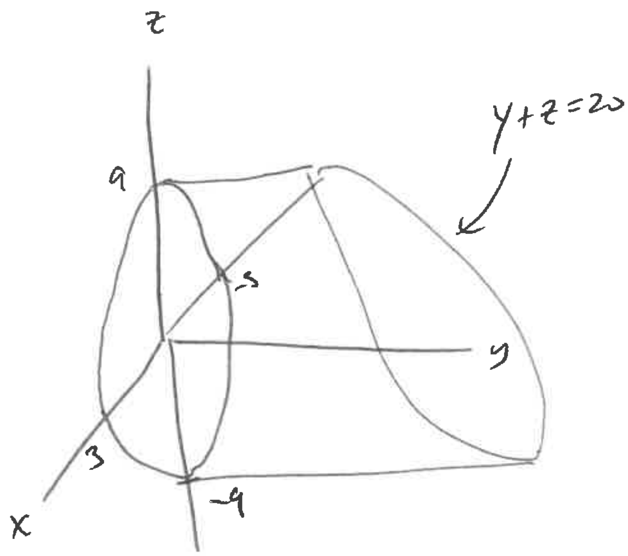
$0 \le x \le 4y$

$$\int_{y=0}^{2} \int_{x=0}^{4y} \frac{\cos(y)}{y}\, dxdy = \int_{y=0}^{2} \frac{x\cos(y)}{y}\Big|_0^{4y} dy = \int_{y=0}^{2} \frac{4y\cos(y)}{y}\, dy$$

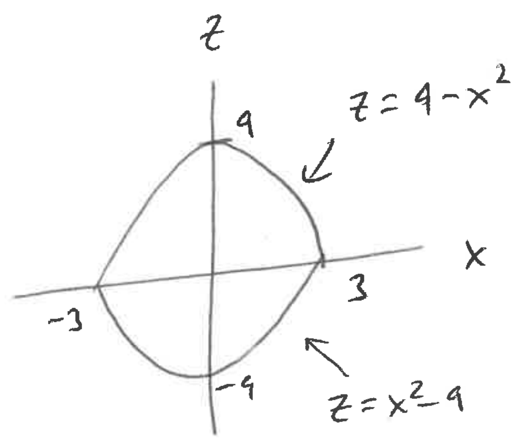
$$= \int_{y=0}^{2} 4\cos(y)\, dy = 4\sin(y)\Big|_0^2 = 4\sin(2) - 4\sin(0)$$

$$= 4\sin(2)$$

4. (10 points) <u>Set up</u> the integral used to find the volume of the solid enclosed by $z = x^2 - 9$, $z = 9 - x^2$, $y = 0$, and $y + z = 20$.



project on xz plane



Left $y = 0$

Right $y = 20 - z$

$-3 \le x \le 3$

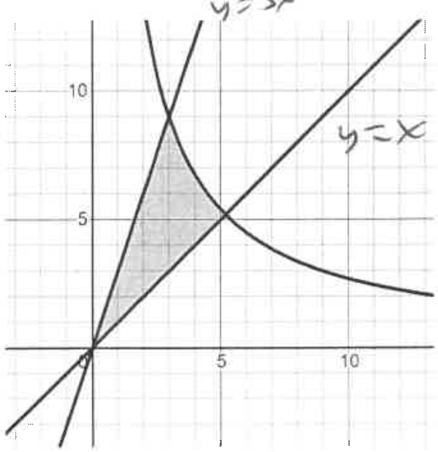
$x^2 - 9 \le z \le 9 - x^2$

$$\int_{x=-3}^{3} \int_{z=x^2-9}^{9-x^2} \int_{y=0}^{y=20-z} 1 \, dy \, dz \, dx$$

5. (11 points) R is the region enclosed by the lines $y = 3x$, $y = x$, and the hyperbola $xy = 27$.

Use the given transformation to change the variables for the given integral. You only have to give the new integral. You do not need to compute the new integral.

$$\iint_R (5x)\,dA \qquad x = \frac{v}{u} \qquad y = u$$



Region R

$$J = \begin{vmatrix} -\frac{v}{u^2} & \frac{1}{u} \\ 1 & 0 \end{vmatrix} = 0 - \frac{1}{u}$$

$$|J| = \frac{1}{u}$$

$\underline{y = x}$

$u = \frac{v}{u}$

$u^2 = v$

$\underline{y = 3x}$

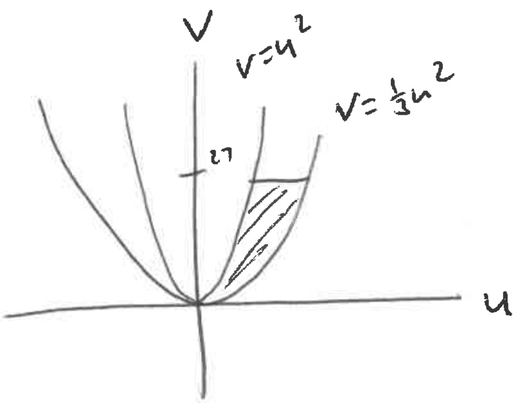
$u = 3\frac{v}{u}$

$\frac{1}{3}u^2 = v$

$xy = 27$

$\frac{v}{u} \cdot u = 27$

$v = 27$



$0 \le v \le 27$

$\sqrt{v} \le u \le \sqrt{3v}$

$$\int_{v=0}^{27} \int_{u=\sqrt{v}}^{\sqrt{3v}} \frac{5v}{u} \cdot \frac{1}{u} \, du\, dv$$

6. (10 points) Set up the integral in **polar** that will find the surface area of the function $z = x^3 + y^2$ above the region on the $xy-$plane that is inside the circle $x^2 + y^2 = 6x$

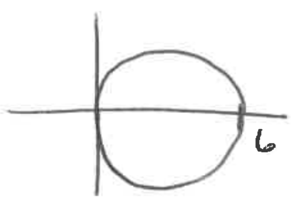
$$SA = \iint_D \sqrt{(z_x)^2 + (z_y)^2 + 1}\, dA = \iint_D \sqrt{(3x^2)^2 + (2y)^2 + 1}\, dA$$

$$= \iint_D \sqrt{9x^4 + 4y^2 + 1}\, dA$$

$x^2 + y^2 = 6x$

$r^2 = 6r\cos\theta$

$r = 6\cos\theta$



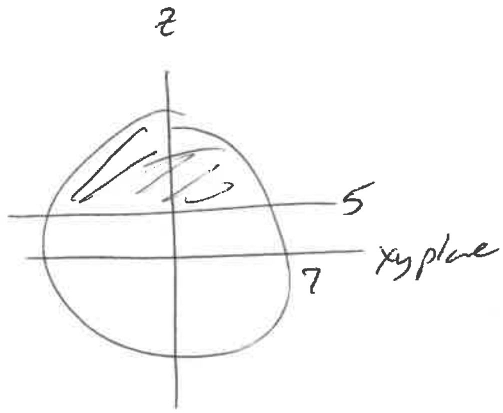
$$\int_{\theta=-\frac{\pi}{2}}^{\pi/2} \int_0^{6\cos\theta} \sqrt{9(r\cos\theta)^4 + 4(r\sin\theta)^2 + 1}\; r\,dr\,d\theta$$

$-\frac{\pi}{2} \le \theta \le \frac{\pi}{2}$

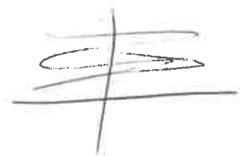
$0 \le r \le 6\cos\theta$

7. (10 points) The solid E is the region that is within the sphere $x^2 + y^2 + z^2 = 49$ and above the plane $z = 5$.
The solid E is also in the half-space $y \geq 0$.

Set up the integral that would find the volume of the solid E.



also $y \geq 0$



$0 \leq \theta \leq \pi$

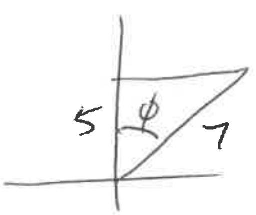
$5\sec\phi \leq \rho \leq 7$

$0 \leq \phi \leq \arccos\left(\frac{5}{7}\right)$

$z = 5$

$\rho\cos\phi = 5$

$\rho = 5\sec\phi$



$\cos\phi = \frac{5}{7}$

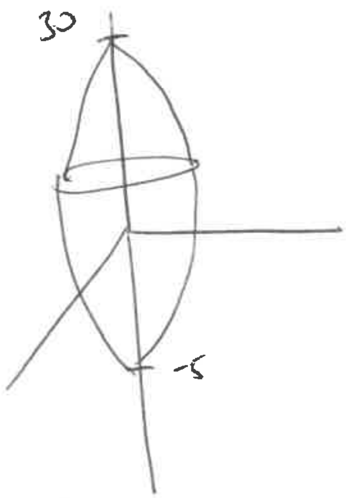
$\phi = \arccos\left(\frac{5}{7}\right)$

$$V = \iiint_E 1\, dV$$

$$= \int_{\theta=0}^{\pi} \int_{\phi=0}^{\arccos(\frac{5}{7})} \int_{\rho=5\sec\phi}^{7} \rho^2 \sin\phi \, d\rho\, d\phi\, d\theta$$

8. (10 points) The solid E is the region that is bounded by $z = 4x^2 + 4y^2 - 5$ and $z = 30 - 3x^2 - 3y^2$ and has $y \geq 0$.

<u>Set up</u> the integral $\iiint_E x\, dV$.



Top $z = 30 - 3x^2 - 3y^2 = 30 - 3r^2$

Bottom $z = 4x^2 + 4y^2 - 5 = 4r^2 - 5$

<u>Intersection</u>

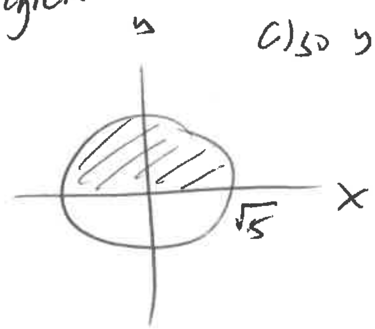
$4x^2 + 4y^2 - 5 = 30 - 3x^2 + 3y^2$

$7x^2 + 7y^2 = 35$

$x^2 + y^2 = 5$

Region D

also $y \geq 0$



$0 \leq \theta \leq \pi$

$0 \leq r \leq \sqrt{5}$

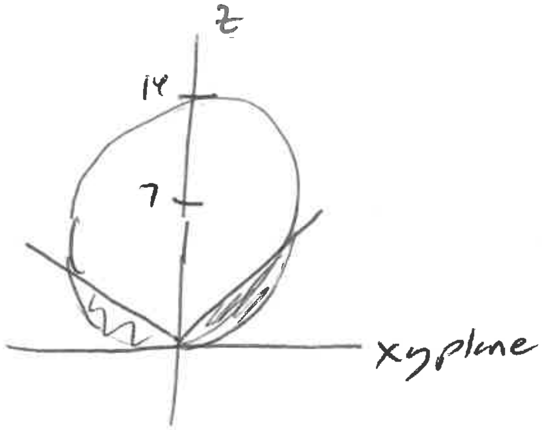
$$\int_{\theta=0}^{\pi} \int_{r=0}^{\sqrt{5}} \int_{z=4r^2-5}^{30-3r^2} r\cos\theta \cdot r \, dz\, dr\, d\theta$$

9. (11 points) The solid E is the region that is within the sphere $x^2 + y^2 + z^2 = 14z$ and below the cone $z = \sqrt{3x^2 + 3y^2}$.
The solid E is also in the half-space $y \geq 0$.

Set up the integral $\iiint_E x \, dV$.

$\hookrightarrow \rho^2 = 14\rho\cos\phi$

$\rho = 14\cos\phi$



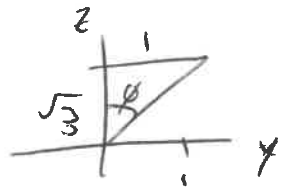
cone

$z = \sqrt{3x^2 + 3y^2}$

let $x = 0$

$z = \sqrt{3}\, y$ let $y = 1$

$z = \sqrt{3}$



$\tan\phi = \frac{1}{\sqrt{3}}$

$\phi = \frac{\pi}{6}$

$0 \leq \theta \leq \pi$

$0 \leq \rho \leq 14\cos\phi$

$\frac{\pi}{6} \leq \phi \leq \frac{\pi}{2}$

$$\int_{\theta=0}^{\pi} \int_{\phi=\frac{\pi}{6}}^{\pi/2} \int_{\rho=0}^{14\cos\phi} \rho\sin\phi\cos\theta \quad \rho^2\sin\phi \, d\rho \, d\phi \, d\theta$$

10. (10 points) Rewrite the integral in the order of **dy dz dx**.

$$\int_{y=0}^{9}\int_{x=-\sqrt{y}}^{\sqrt{y}}\int_{z=0}^{36-4y} f(x,y,z)\,dz\,dx\,dy$$

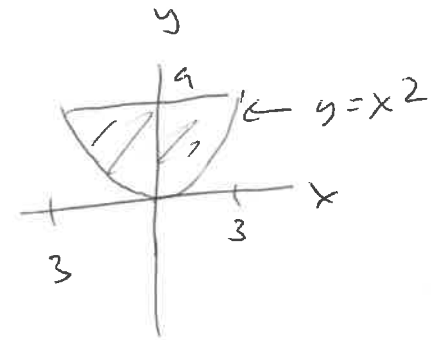
Top $z=36-4y$

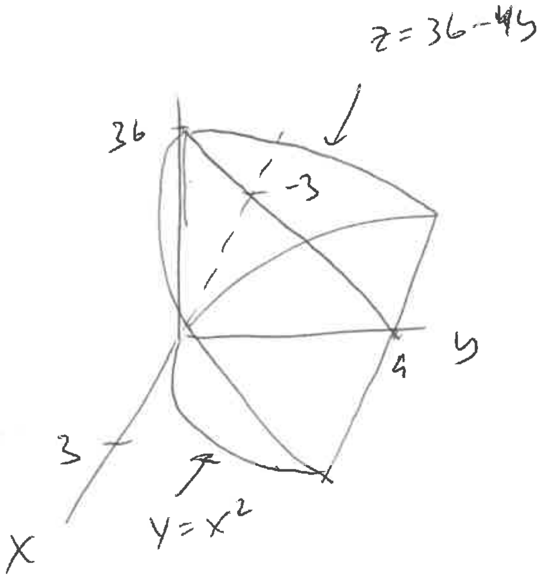
Bottom $z=0$

Region D

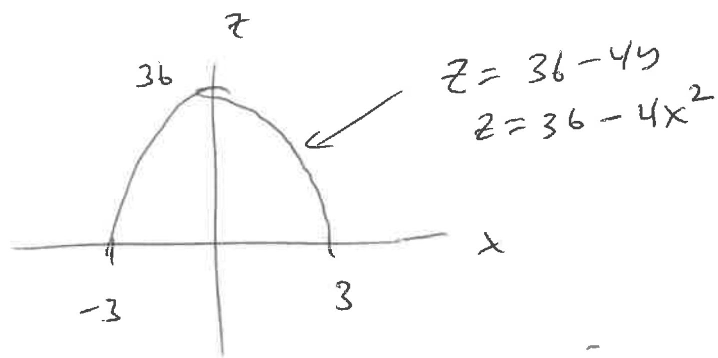
$-\sqrt{y} \le x \le \sqrt{y}$

$0 \le y \le 9$





project on zx plane



dz dx ↑ $0 \le z \le 36-4x^2$

$-3 \le x \le 3$

Left $y=x^2$

Right $y=\dfrac{36-z}{4}$

$$\int_{x=-3}^{3}\int_{z=0}^{36-4x^2}\int_{y=x^2}^{\frac{36-z}{4}} f(x,y,z)\,dy\,dz\,dx$$