

The solutions for this quiz **MUST** be submitted in gradescope by 11:55pm on Thursday, January 25, 2024.

You will be graded on both the correct answer and the correctness of the work that you provide to justify that answer. I expect to see all of your work in a neat and orderly manner.

You are allowed to use your class notes and a calculator when working the quiz. However, I would suggest trying to do quiz without using any other resources to see if you actually know the material. You are not allowed to ask other people for help with the questions. If you need clarification on a question, send me an e-mail or ask me in person.

You do not need to turn in this cover sheet when you submit your work. You are expected to tell webassign where on what pages your questions are located.

I would suggest setting an alarm on your phone so that you do not forget to submit the quiz. You can submit multiple times to gradescope. Only the last submission is graded.

- (2 points) How far is the center of the sphere from the following?  
sphere:  $x^2 - 2x + y^2 - 4y + z^2 - 6z = 6$ 
  - The  $xy$ -plane.
  - The  $z$ -axis.
- (2 point) You have the points  $A(1, 3, 6)$  and  $B(2, 7, 2)$ . Find a vector of length 3 that is in the direction of  $\overline{BA}$ .
- (2 points) Let  $\mathbf{c} = \langle 1, 2, 4 \rangle$  and  $\mathbf{d} = \langle 2, 0, 1 \rangle$ . Find  $\cos \theta$  where  $\theta$  is the angle between vectors  $\mathbf{c}$  and  $\mathbf{d}$ . do not give a decimal answer.
- (2 points) Let  $\mathbf{c} = \langle 2, -5, 1 \rangle$  and  $\mathbf{d} = \langle 2, 1, 3 \rangle$ . Find the scalar projection of  $\mathbf{d}$  onto  $\mathbf{c}$ . do not give a decimal answer.
- (2 points) Let  $\mathbf{c} = \langle 2, -5, 1 \rangle$  and  $\mathbf{d} = \langle 2, 1, 3 \rangle$ . Find the vector projection of  $\mathbf{d}$  onto  $\mathbf{c}$ . do not give a decimal answer.

# Quiz 1      Key

$$1) \quad x^2 - 2x + y^2 - 4y + z^2 - 6z = 6$$

$$x^2 - 2x + 1^2 + y^2 - 4y + 2^2 + z^2 - 6z + 3^2 = 6 + 1^2 + 2^2 + 3^2$$

$$(x-1)^2 + (y-2)^2 + (z-3)^2 = 6 + 1 + 4 + 9 = 20$$

center  $(1, 2, 3)$       radius  $\sqrt{20}$

A) distance from  $xy$  plane is 3

B) distance from  $z$ -axis is distance from the points  $(1, 2, 3)$  to  $(0, 0, 3)$

$$= \sqrt{(1-0)^2 + (2-0)^2 + (3-3)^2}$$

$$= \sqrt{1 + 4}$$

$$= \underline{\underline{\sqrt{5}}}$$

2]

$$\vec{BA} = \langle 1-2, 3-7, 6-2 \rangle$$

$$= \langle -1, -4, 4 \rangle$$

$$|\vec{BA}| = \sqrt{(-1)^2 + (-4)^2 + 4^2} = \sqrt{1 + 16 + 16}$$

$$= \sqrt{33}$$

$$\text{Answer} = \frac{3}{\sqrt{33}} \langle -1, -4, 4 \rangle \quad \text{or} \quad \left\langle \frac{-3}{\sqrt{33}}, \frac{-12}{\sqrt{33}}, \frac{12}{\sqrt{33}} \right\rangle$$


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3]

$$c \cdot d = |c| |d| \cos \theta$$

$$\cos \theta = \frac{c \cdot d}{|c| |d|}$$

$$= \frac{1(2) + 2(0) + 4(1)}{\sqrt{21} \sqrt{5}}$$

$$\cos \theta = \frac{6}{\sqrt{21} \sqrt{5}}$$

$$|c| = \sqrt{1^2 + 2^2 + 4^2}$$

$$= \sqrt{1 + 4 + 16} = \sqrt{21}$$

$$|d| = \sqrt{2^2 + 0 + 1^2} = \sqrt{5}$$

$$\text{or } \left\langle \frac{1}{2}, -\frac{1}{5}, \frac{1}{15} \right\rangle$$

$$\text{or } \frac{1}{15} \langle 2, -5, 1 \rangle$$

$$= \frac{30}{2} \langle 2, -5, 1 \rangle$$

$$= \frac{(\sqrt{30})}{2} \langle 2, -5, 1 \rangle$$

$$\boxed{5} \quad \text{proj}_c = \frac{c \cdot d}{|c|^2} c$$

$$= \frac{\sqrt{30}}{4-5+3} = \frac{\sqrt{30}}{2}$$

$$= \frac{\sqrt{30}}{2(2)+(-5)(1)+1(3)}$$

$$\boxed{4} \quad \text{comp}_c d = \frac{c \cdot d}{|c|}$$

$$|c| = \frac{\sqrt{2^2+(-5)^2+1^2}}{\sqrt{4+25+1}} = \sqrt{30}$$