Quiz #4 Key Math 251 February 22, 2024

The solutions for this quiz MUST be submitted in gradescope by 11:55pm on Thursday, February 22, 2024.

You will be graded on both the correct answer and the correctness of the work that you provide to justify that answer. I expect to see all of your work in a neat and orderly manner.

You are allowed to your your class notes and a calculator when working the quiz. However, I would suggest trying to do quiz without using any other resources to see if you actually know the material. You are not allowed to ask other people for help with the questions. If you need clarification on a question, send me an e-mail or ask during office hours.

You do not need to turn in this cover sheet when you submitt your work. You are expected to tell webassign where on what pages your questions are located.

I would suggest setting an alarm on your phone so that you do not forget to submit the quiz.
You can submitt multiple time to gradescope. Only the last submission is graded.
DO NOT DO YOUR WORK ON THIS PAPER.

1. (2 points) Let $F(x, y, z) = x^2e^{2z} + 4y^3 + y^2z^3$. Compute ∇F

2. (3 points) Compute $\dfrac{\partial z}{\partial x}$ for the equation.

   $x^4 + y^2e^z = 5z^2 + x^2y^4$

3. (2 points) Find the directional derivative of the function at the given point in the direction of the vector $\mathbf{v}$.

   $f(x, y) = x^2y^3 + 4yx^3$ $\quad\quad P(1, 2) \quad\quad \mathbf{v} = \langle 1, 5\rangle$

4. (3 points) Find the linear approximation of the function at the indicated point.

   $f(x, y) = \sqrt{29 - 3x^2 - y^2}$ at $(2, 1)$

Quiz 4 Key

1) $\nabla F = \langle F_x, F_y, F_z \rangle$

$= \langle 2xe^{2z}, 12y^2 + 2yz^3, 2x^2e^{2z} + 3y^2z^2 \rangle$

2) $\underbrace{x^4 + y^2e^z - 5z^2 - x^2y^4}_{F(x,y,z)} = 0$

$$\frac{\partial z}{\partial x} = z_x = \frac{-F_x}{F_z} = \frac{-(4x^3 - 2xy^4)}{y^2e^z - 10z}$$

3) $\nabla f = \langle f_x, f_y \rangle = \langle 2xy^3 + 12yx^2, 3x^2y^2 + 4x^3 \rangle$

$\nabla f(1,2) = \langle 2(2)^3 + 12(2), 3(2)^2 + 4 \rangle$

$= \langle 40, 16 \rangle$

$|v| = \sqrt{1^2 + 5^2} = \sqrt{26}$ $\qquad u = \frac{1}{\sqrt{26}} \langle 1, 5 \rangle$

$D_u f(1,2) = \nabla f(1,2) \cdot u = \frac{1}{\sqrt{26}} [40 + 16(5)] = \frac{1}{\sqrt{26}}(120)$

$= \frac{120}{\sqrt{26}}$

4) $f(2,1) = \sqrt{29-3(4)-1} = \sqrt{29-13} = \sqrt{16} = 4$

$$f_x = \frac{1}{2}\left(29-3x^2-y^2\right)^{-1/2} \cdot (-6x)$$

$$f_x = \frac{-3x}{\sqrt{29-3x^2-y^2}}$$

$$f_y = \frac{1}{2}\left(29-3x^2-y^2\right)^{-1/2} (-2y)$$

$$f_y = \frac{-y}{\sqrt{29-3x^2-y^2}}$$

$f_x(2,1) = \dfrac{-6}{4}$ $\qquad$ $f_y(2,1) = \dfrac{-1}{4}$

$$L(x,y) = f(2,1) + f_x(2,1)(x-2) + f_y(2,1)(y-1)$$

$$= 4 + \frac{-6}{4}(x-2) + \frac{-1}{4}(y-1)$$

$$= 4 - \frac{6}{4}(x-2) - \frac{1}{4}(y-1)$$ ← great answer

$$= \frac{29}{4} - \frac{3x}{2} - \frac{y}{4}$$