Math 325 . Exam 2 Friday, March 8, 2024

Printed Name: ______________________

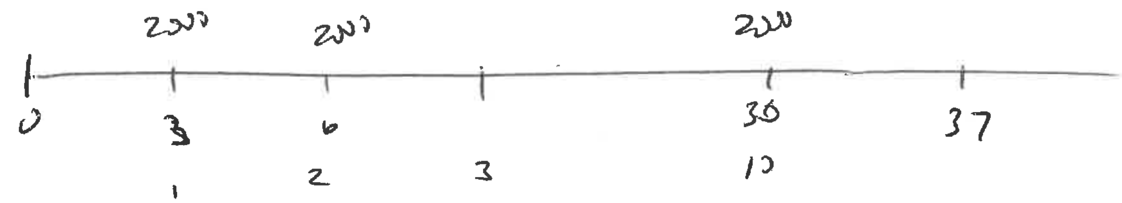
Signature: Key

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

- Point values for each problem are as indicated.
- **To receive full credit for each problem, you must show all appropriate work, and your work must be presented in a clear, organized manner that is easy to follow.**
- If you need more space to work a problem, you may use the back of the exam. Please indicate where the problem is located.
- You may use up to two calculators on the exam.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Check the back of the page for problem.

1. (12 points) Ten payments of \$2,000 each are made at three-year intervals into an account. Find the accumulated value of the account 34 years after the first payment. Assume $i^{(2)} = 4\%$.



let j = 3 yr eff. Rate

$$1+j = \left(1+\frac{i^{(2)}}{2}\right)^{6}$$

$$j = 12.61624\%$$

$$2000\, s_{\overline{10}|j}\left(1+\frac{i^{(2)}}{2}\right)^{14}$$

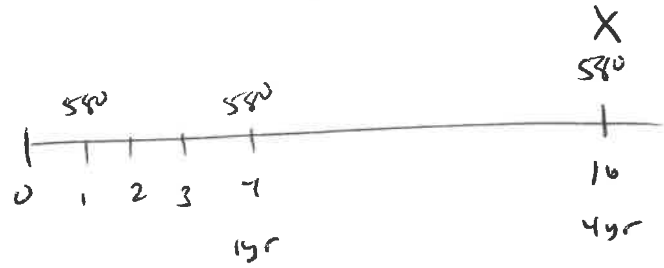
$$= (36160.22587)(1.02)^{14}$$

$$= \$47{,}712.65$$

2. (12 points) Mason borrows \$8,000 from a bank that charges him compound interest at a nominal annual rate of 8% compounded quarterly. The bank requires him to make a payment of \$580 at the end of each quarter for four years, with the last payment being a balloon payments. Find the amount of the balloon payment.

$i^{(4)} = 8\%$

$\frac{i^{(4)}}{4} = 2\%$



$$8000 = 580\, a_{\overline{16}|2\%} + X v^{16}$$

$$8000 - 580\, a_{\overline{16}|} = X v^{16}$$

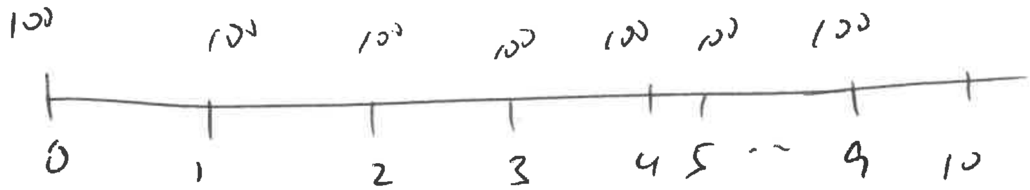
$$\frac{8000 - 7875.07}{v^{16}} = X$$

$$X = \frac{124}{(1.02)^{-16}} = 124(1.02)^{16}$$

$$= 171.50$$

$$\text{Balloon pmt} = X + 580 = 751.50$$

3. (14 points) Pam deposits 100 into an account at the beginning of each year. The account credits interest at an annual effective interest rate i. The accumulated amount in the account after 5 years is X and the accumulated amount in the account after 10 years is $5X$. Calculate X.



i = annual eff.

after 5yrs

$$X = 100\ddot{s}_{\overline{5}|i}$$

after 10 yrs.

$$5X = 100\ddot{s}_{\overline{10}|i}$$

$$5 \cdot 100\ddot{s}_{\overline{5}|i} = 100\ddot{s}_{\overline{10}|}$$

$$5 \cdot \frac{(1+i)^5 - 1}{d} = \frac{(1+i)^{10} - 1}{d}$$

$$5(1+i)^5 - 5 = (1+i)^{10} - 1$$

$$0 = (1+i)^{10} - 5(1+i)^5 + 4$$

$(1+i)^5 = A$

$$0 = A^2 - 5A + 4$$

$$0 = (A-4)(A-1)$$

$A = 4$; $(1+i)^5 = 4$; $i = 4^{1/5} - 1$

$A = 1$; $(1+i)^5 = 1$; $\Rightarrow i = 0$; X

$i = .3195079$

$$X = 100 \cdot \frac{(1+i)^5 - 1}{d}$$

$$X = 100 \cdot \frac{4-1}{\frac{i}{1+i}}$$

$$X = 100 \cdot \frac{3}{4^{1/5} - 1} \cdot 4^{1/5}$$

$$X = 1238.9438$$

4. (12 points) \$32,000 is deposited into a fund which is accumulating at 5% per year convertible continuously. If money is withdrawn continuously at the rate of \$1,600 per annum, how long will the fund last? Give your answer to at lest 3 decimal digits.

$\delta = 5\%$

$$32000 = 1600\, \bar{a}_{\overline{n}|\, \delta = 5\%}$$

$$32000 = 1600 \frac{1 - e^{-.05n}}{.05}$$

$$20 = \frac{1 - e^{-.05n}}{.05}$$

$$20(.05) = 1 - e^{-.05n}$$

$$e^{-.05n} = 1 - 20(.05) = 1 - 1 = 0$$

$$e^{-.05n} = 0$$

Can not be solved.

no solution.

The original problem had the intial deposit being 30,000.

using this value and solving for n gives

$n = 55.45$ yrs.

I changed the 30000 to 32000 so the division $\frac{32000}{1600}$ was a nice number.

problem was not graded.
everybody got full credit.

5. (12 points) Find the present value of a 40 year annuity-immediate with annual payments in which the first payment is \$200, the second payment is \$220, the third payment is \$242, the fourth payment is \$266.20, etc., assuming an annual effective rate of interest of 6%.

time	pmt
0	—
1	200
2	$220 = 200(1.10)$
3	$242 = 200(1.10)^2$
4	$266.20 = 200(1.10)^3$

geometric annuity.

$K = 10\%$

$i = 6\%$

$n = 40$

$$PV = 200\left[\frac{1-\left(\frac{1+K}{1+i}\right)^n}{i-K}\right]$$

$$= 200\left[\frac{1-\left(\frac{1.10}{1.06}\right)^{40}}{.06-.10}\right]$$

$$= 200(85.005096)$$

$$= 17001.019$$

6. (12 points)David purchases a perpetuity immediate that earns an annual effective rate of interest of 10%. The perpetuity has an initial annual payment of \$250 and each successive payment decreases by 4%. What was the purchase price of this annuity?

$i = 10\%$ $\quad K = -4\%$

$$PV = \frac{pmT}{i-K} = \frac{250}{.10 - -.04} = \frac{250}{.14}$$

$$= 1785.71$$

7. (12 points) Compute the value of a perpetuiry-due that has semiannual payments with the first payment of \$50 now and each payment increases by \$20. Assumme that $i^{(2)} = 12\%$.

$P = 50$ $\quad Q = 20$ $\quad j = \frac{i^{(2)}}{2} = 6\%$ $\quad d = \frac{j}{1+j} = \frac{.06}{1.06}$

$$PV = \left(\frac{P}{i} + \frac{Q}{i^2}\right)(1+i) \quad \text{or} \quad PV = \frac{P}{d} + \frac{Q}{id}$$

$$= \left[\frac{50}{.06} + \frac{20}{(.06)^2}\right](1.06)$$

$$= (6388.8889)(1.06)$$

$$= 6772.22$$

8. (14 points) Jacob purchases a perpetuity-immediate for 3,000 which has annual payments of 120 and annual effective rate i.

One the same day and at the same price and with an annual effective rate i, Mark purchased an annuity with 20 annual payments. The first payments will be 5 years from today and will have an amount \$55 and each subsequent payment is X dollars more than the previous payment.

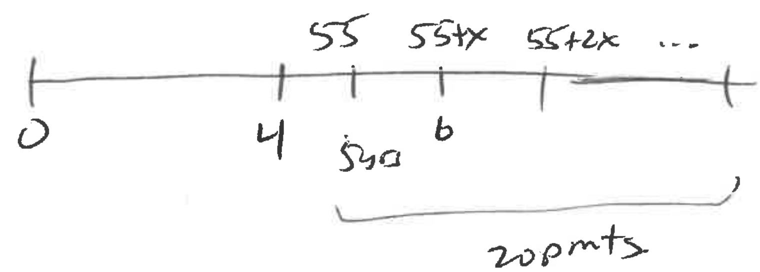
Calculate X.

Jacob.

$PV = 3000 = \frac{120}{i}$

$i = \frac{120}{3000} = .04$

mark.



mark.

$$3000 = \left[55\, a_{\overline{20|}i} + X\, \frac{a_{\overline{20|}} - 20v^{20}}{i}\right] v^4$$

$$3000(1+i)^4 - 55\, a_{\overline{20|}} = X\, \frac{a_{\overline{20|}} - 20v^{20}}{i}$$

$$3509.57568 - 55(13.590326) = X\, \frac{13.590326 - 20(1.04)^{-20}}{.04}$$

$$2762.1077 = X(111.5646855)$$

$$X = 24.7579$$

$$X = 24.76$$