Math 325 Exam 3 Monday, April 8, 2024

Printed Name: Key

Signature: _____

On my honor, as an Aggie, I have neither given nor received unauthorized aid on this academic work.

- Point values for each problem are as indicated.
- **To receive full credit for each problem, you must show all appropriate work, and your work must be presented in a clear, organized manner that is easy to follow.**
- If you need more space to work a problem, you may use the back of the exam. Please indicate where the problem is located.
- You may use up to two calculators on the exam.
- SCHOLASTIC DISHONESTY WILL NOT BE TOLERATED.

Check the back of the page for problem.

1. (12 points) A loan is being repaid with 60 monthly payments of \$1,000 each. The nominal annual rate of interest for this loan was 12% compounded monthly.
At the time of the 20th payment, the borrower wishes to pay an additional \$10,000. Then the outstanding balance will be refinanced over the next 2 years with a new nominal annual rate of interest of 7% convertible monthly.
Compute the amount of the revised monthly payment.

Ballance $1000 a_{\overline{40}|.01} = 32834.69$

Pay additional 10000 gives Ballance 22,834.69

New loan. $22834.69 = R a_{\overline{24}|\frac{7}{12}\%}$

TVM

$N = 24$

$I = 7/12$

$PV = 22834.69$

$PMT =$ solve

$FV = 0$

new pmt is 1022.367

2. (10 points) Anthony borrowed \$20,000 from the bank. The loan has an annual effective rate of 5% and payments will be made at the end of each year. Anthony's first payment was \$1,500 and future payments will increase by \$200 each year. The last payment of the loan will be a drop payment (a smaller additional payment that is made one period after the last regular payment). What is the balance of the loan at the end of the 8th year?

$IS_{\overline{8}|5\%} = 9.549108876$

Retrospective method.

$$20000(1.05)^8 - \left[1500 s_{\overline{8}|5\%} + 200 \frac{s_{\overline{8}|} - 8}{.05}\right]$$

$$29549.1088 - \left[14323.6633 + 6196.435\right]$$

Ballance = 9029.01

Check the back of the page for problem.

3. Matt buys a house and finances the loan over k years with level monthly payments and interest charged at 4% per year compounded monthly. If the first payment consists of \$629.10 of principal, and the 100th payment consists of \$438.72 of interest,

(a) (7 points) what is the monthly payment? Round to 2 decimal places.

$P_1 = 629.10$

$I_{100} = 438.72$

$P_{100} = P_1\left(1+\frac{.04}{12}\right)^{99} = 874.577$

$= 874.58$

$R = P_{100} + I_{100} = 1313.297$

$= 1313.30$

(b) (7 points) what is the original amount of the loan?

$P_1 = R - I_1$

$I_1 = R - P_1 = 1313.30 - 629.10$

$= 684.20$

$I_1 = B_0\left(\frac{.04}{12}\right) \Rightarrow B_0 = I_1\left(\frac{12}{.04}\right) = \$205,260$

B_0 = Loan Amt.

Check the back of the page for problem.

4. (12 points) Jake borrows \$10,000 and is going to pay off the loan with a sinking fund at the end of 10 years.

   Service on the loan:
   first 6 years: 10% annual effective
   last 4 years: 5% annual effective

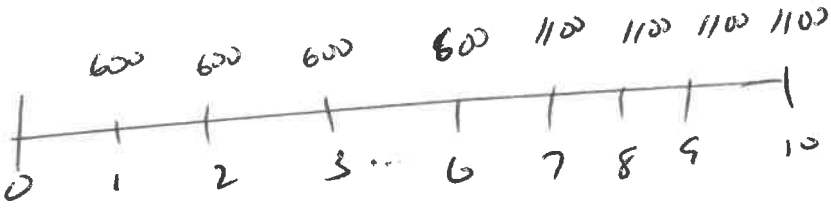
   Sinking fund:
   7% annual effective

   Jake will make annual payments of \$1,600. NOTE: These payments of \$1,600 will be split to pay both the service on the loan with the remaining money being deposited into the sinking fun.

   Determine if the sinking fund had the necessary fund to pay off the loan. Give the amount of extra money in the account or the amount of additional funds needed.

Term	Service	sinking fund deposit
1st 6 yrs	1000	600
last 4 yrs	500	1100

Sinking fund



Sinking fund end Ballance

$600 s_{\overline{10}|7\%} + 500 s_{\overline{4}|7\%}$

$8289.87 + 2219.97 = 10509.84$

509.84 extra dollars.

Check the back of the page for problem.

5. (8 points) A k year bond had level annual coupons and an annual effective yield of 7.2%. The bond has the following book values: determine the amount of the annual coupon and also determine if the bond sold at a premium or sold at a discount.

$B_8 = 1,130.56$ $B_9 = 1,127.96$

(A) Determine the amount of the annual coupon of the bond.
(B) Was this bond sold at a premium or a discount? Or can this not be determined?

$B_9 = B_8(1+i) - F_r$

$F_r = B_8(1+i) - B_9$

$= 1211.96 - 1127.96$

$= 84$

Since $B_8 > B_9$

The value of the Bond is going down as time passes "writting down" the bond. Thus it was sold at a premium

6. (10 points) A 20-year bond with 6% semiannual coupons and a par value of \$100 is purchased for \$89.50 when the bond was issued. Compute the nominal yield convertible semiannually for this bond. Give your answer to three decimal places.

$n =$ $F = 100 = C$ $F_r = 3$

$r = 3\%$ price $= 89.50$

TVM Solver

N=
I= solve ⟶ $3.491\% = i^{(2)}/2$
PV = −89.50
PMT = 3
FV = 100

$i^{(2)} = 6.982\%$

Check the back of the page for problem.

7. (12 points) A \$2,000 par value bond with 10% semiannual coupons. The bond is bought to yield 6% semiannually. The bond has a redemption value of \$2,100 and will mature on September 7, 2030. Coupons are paid on March 7 and September 7.

   Suppose the bond is bought by a new owner on June 17, 2024. Using the practical method, find the flat price and accrued coupon of the bond on that date. Assume that you are using the actual number days.

$i = 3\%$ $\quad Fr = 2000(.05) = 100$ $\quad C = 2100$ $\quad$ mature 9/7/30

Bond value on mrch 7, 2024 $\quad$ Remaining coupons: 13

$B = 100\, a_{\overline{13}|3\%} + 2100\, v^{13} = 2493.49$

Value on 6/17/24

$K = \dfrac{dbd(3.0724, 6.1724, ACT)}{dbd(3.0724, 9.0724, ACT)} = \dfrac{102}{184}$

$B = 2493.49\left(1+(.03)\frac{102}{184}\right) = 2534.96$

$Fr_{IC} = 100\left(\frac{102}{184}\right) = 55.4347$

8. (10 points) A \$1,000 bond with a coupon rate of 6.4% payable semiannually is redeemable after an unspecified number of years at \$1,200. The bond is bought to yield 10% convertible semiannually. If the present value of the redemption value is \$372 at this yield rate, find the purchase price.

$r = 3.2$ $\quad n = ??$ $\quad i = 5\%$ $\quad Fr = Cg$

$F = 1000$ $\quad C = 1200$ $\quad K = 372$ $\quad \dfrac{1000(.032)}{1200} = g$

$g = \dfrac{32}{1200} = \dfrac{2}{75}$

$P = K + \dfrac{g}{i}(C-K)$

$= 372 + \dfrac{2/75}{.05}(1200-372)$

$P = 813.6$

9. (12 points) Consider a ten year, \$1000 par value 8% bond with annual coupons called at \$1050 on any coupon date starting after the coupon is paid at the end of the 5th year and later callable at \$1000 starting after the coupon is paid at the end of the 8th year and later.

Find the highest price which an investor can pay and still be certain of a yield of 7% convertible semiannually.

$F=1000$ $r=.08$ $i=7\%$ $i^{(2)}=7\%$ $i=7.1225$

$C_1=1050$ $g=\frac{Fr}{C}=\frac{1000(.08)}{1050}=\frac{80}{1050}=7.619\% > i$ premium

price at $n=5$

TVM $n=5$, $i=7.1225\%$, $PV=$ solve., $PMT=80$, $FV=1050$ price $=1071.31$

$C=1000$ $g=\frac{Fr}{C}=\frac{80}{1000}=8\% > i$ premium price at $n=8$

TVM $N=8$, $i=7.1225\%$, $PMT=80$, $FV=1000$, $PV=$ solve price $=1052.15$

Answer \$1052.15

Check the back of the page for problem.