Here are the critical values of the function $f(x)$ and the second derivative. What can be said about the critical values?

1. $f'' = 2x + 2$, cv: $x = -5, 3$
2. $f'' = -6x - 6$, cv: $x = -5, 3$
3. $f'' = 42x - 36x^2$, cv:$x = 0, 2$
4. $f'' = (x^2 + 4x + 2)e^x$, cv: $x = -2, 0$

Here is a function, its first and second derivative. Do the following steps.

A) Find the domain.
B) Find the intercepts. (if they are easy to compute)
C) Find the asymptotes.
D) Find the first derivative information.
Critical values.
Intervals of increasing and decreasing.
Classify the critical values.
E) Find the second derivative information.
Possible inflection values.
Intervals where concave up and concave down.
Any Inflection points?
F) Use the above information to sketch a graph.

5. $y = \dfrac{-2}{x^2 - x - 6}$

   $y' = \dfrac{2(2x-1)}{(x^2 - x - 6)^2}$

   $y'' = \dfrac{-4(3x^2 - 3x + 7)}{(x^2 - x - 6)^3}$

6. $y = x + \dfrac{9}{x}$

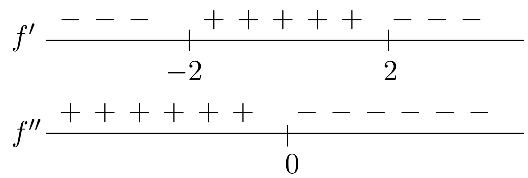
   $y' = 1 - \dfrac{9}{x^2}$

   $y'' = \dfrac{18}{x^3}$

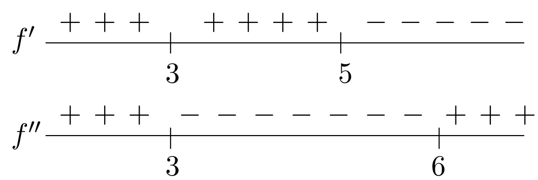
Sketch the graph of a single function that has all of the properties.

7. Continuous for all real numbers.
   Differentiable for all real numbers.
   $f'(-1) = 0$, $f'(1) = 0$
   $f(-1) = 4$, $f(1) = 0$.
   $f'(x) < 0$ on $(-1, 1)$.
   $f'(x) > 0$ on $(-\infty, -1)$ and $(1, \infty)$.
   $f''(x) < 0$ on $(-\infty, 0)$.
   $f''(x) > 0$ on $(0, \infty)$.

8. Continuous for all real numbers.
   Differentiable for all real numbers.
   x-intercepts $0, 4$, and -4.
   $f'(2) = 0$, $f'(-2) = 0$.
   $f''(0) = 0$



9. Continuous for all real numbers except $x = 3$
   Differentiable for all real numbers except $x = 3$
   critical value at $x = 5$
   $\lim_{x\to\infty} f(x) = 0$. $\lim_{x\to-\infty} f(x) = 0$



10. Continuous for all real numbers except $x = -2, 0, 2$
    Differentiable for all real numbers except $x = -2, 0, 2$
    Inflection points at $(-1, 0)$ and $(1, 0)$.
    Vertical Asymptote: $x = -2, x = 2$, and $x = 0$.
    $\lim_{x\to\infty} f(x) = 0$ and $\lim_{x\to-\infty} f(x) = 0$
    $f'(x) < 0$ on $(-\infty, -2)$ and $(-2, 0)$.
    $f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$.
    $f''(x) > 0$ on $(-2, -1)$ and $(1, 2)$.
    $f''(x) < 0$ on $(-\infty, -2)$, $(-1, 0)$, $(0, 1)$, and $(2, \infty)$.

11. Domain: all real numbers except $x = 2$ and $x = -2$
    Continuous for all real numbers except $x = -2, 2$
    Not differentiable at $x = -2, 2$
    x–intercept: 0
    y–intercept: 0
    vertical asymptote: $x = -2$ and $x = 2$
    horizontal asymptote: none
    relative maxima at the point $(4, -4)$
    relative minima at the points $(-4, 4)$
    inflection point: $(0, 0)$
    $f'(x) > 0$ on $(-4, -2)$, $(-2, 2)$, and $(2, 4)$
    $f'(x) < 0$ on $(-\infty, -4)$, and $(4, \infty)$
    $f''(x) > 0$ on $(-\infty, -2)$ and $(0, 2)$
    $f''(x) < 0$ on $(-2, 0)$, and $(2, \infty)$

12. Continuous and differentiable for all real numbers.
$f'(-1) = 0$ and $f'(5) = 0$
$f'(x) > 0$ on $(-1, 5)$ and $(5, \infty)$
$f'(x) < 0$ on $(-\infty, -1)$
$f''(x) > 0$ on $(-\infty, 2)$ and $(5, \infty)$
$f''(x) < 0$ on $(2, 5)$

13. Continuous for all real numbers except $x = 1$ where it has a vertical asymptote.
Differentiable everywhere except at $x = 1$ and $x = 5$
Horizontal asymptote of $y = 0$.
$f'(5)$ =DNE and $f(5) = 4$
$f'(x) < 0$ on $(5, \infty)$
$f'(x) > 0$ on $(-\infty, 1)$ and $(1, 5)$
$f''(x) < 0$ on $(1, 5)$
$f''(x) > 0$ on $(-\infty, 1)$ and $(5, \infty)$

14. Continuous for all real numbers.
Differentiable everywhere except at $x = 0$
Horizontal asymptote of $y = 5$.
$f'(2) = 0$ and $f(2) = 1$
$f'(x) < 0$ on $(-\infty, 0)$
$f'(x) > 0$ on $(0, 2)$ and $(2, \infty)$
$f''(x) < 0$ on $(-\infty, 0)$ and $(0, 2)$ and $(4, \infty)$
$f''(x) > 0$ on $(2, 4)$

15. Continuous for all real numbers.
Differentiable everywhere except at $x = 2$
$\lim_{x\to\infty} f(x) = 3$
$f'(6) = 0$ and $f(6) = 6$
$f''(8) = 0$
$f'(x) < 0$ on $(-\infty, 2)$ and $(6, \infty)$
$f'(x) > 0$ on $(2, 6)$
$f''(x) < 0$ on $(2, 8)$
$f''(x) > 0$ on $(-\infty, 2)$ and $(8, \infty)$

Solutions

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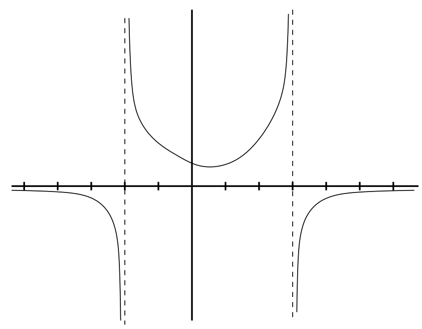
1. Relative maximum at $x = -5$
Relative minimum at $x = 3$

2. relative maximum at $x = 3$
relative minimum at $x = -5$

3. relative maximum at $x = 2$
nothing can be said about $x = 0$

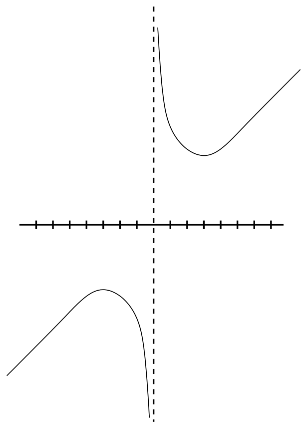
4. relative maximum at $x = -2$
relative minimum at $x = 0$

5. (a) Domain is all real numbers except $x = 3, -2$
(b) y intercept of $\frac{1}{3}$
(c) VA: $x = 3$, $x = -2$
HA: $y = 0$
(d) i. cv: $x = \frac{1}{2}$
ii. inc: $(\frac{1}{2}, 3)$, $(3, \infty)$
dec: $(-\infty, -2)$, $(-2, \frac{1}{2})$
iii. rel min at $x = \frac{1}{2}$
(e) i. no possible inflection values.
ii. c.u.: $(-2, 3)$
c.d.: $(-\infty, -2)$, $(3, \infty)$
iii. no inflection points
(f) Graph of $f(x)$.

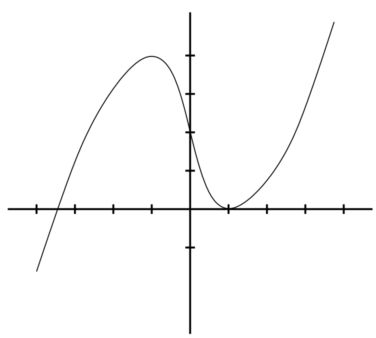


6. (a) Domain is all real numbers except $x = 0$
(b) no intercepts
(c) VA: $x = 0$
HA: none
(d) i. cv: $x = -3, 3$
ii. inc: $(-\infty, -3)$, $(3, \infty)$
dec: $(-3, 0)$, $(0, 3)$
iii. rel min at $x = 3$
rel max at $x = -3$
(e) i. no possible inflection values.
ii. c.u: $(0, \infty)$
c.d.: $(-\infty, 0)$
iii. no inflection points

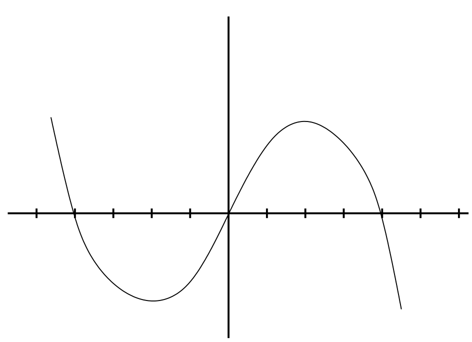
(f) Graph of $f(x)$.



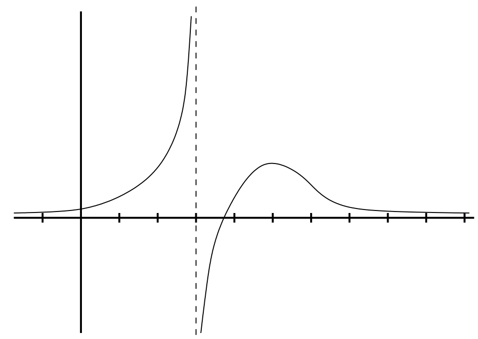
7. Graph of $f(x)$.



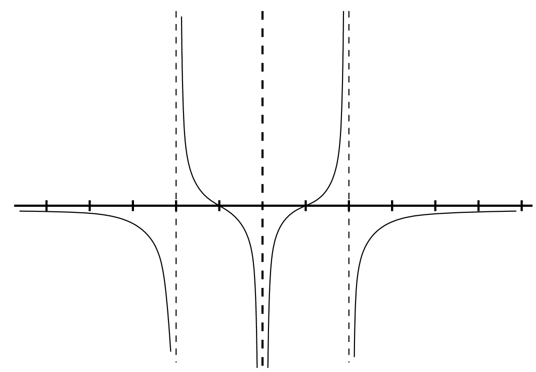
8. Graph of $f(x)$.



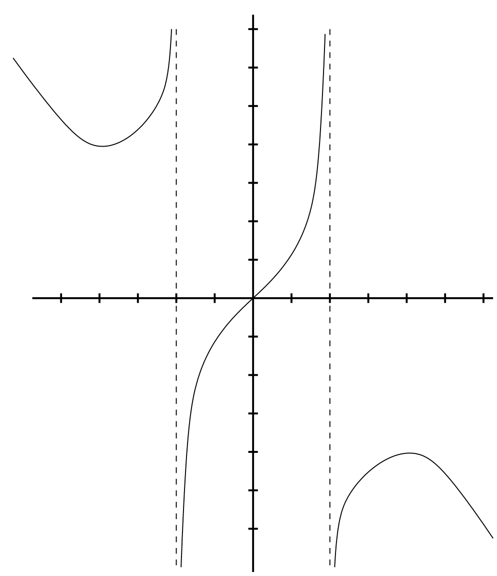
9. Graph of $f(x)$.



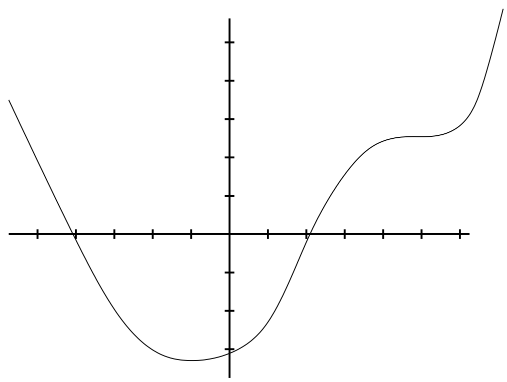
10. Graph of $f(x)$.



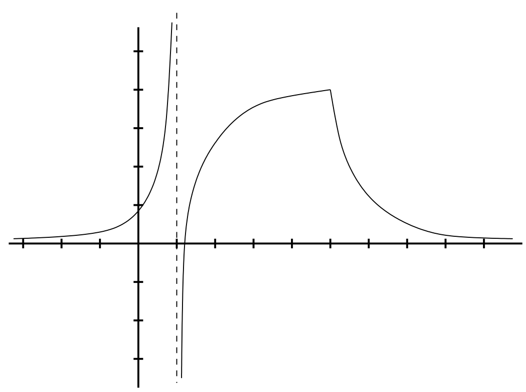
11. Graph of $f(x)$.



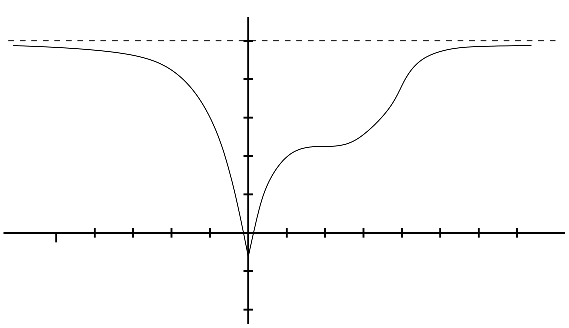
12. Graph of $f(x)$.



13. Graph of $f(x)$.



14. Graph of $f(x)$.



15. Graph of $f(x)$.