

# Concepts to Know #1

Math 141

1.3-1.5, 2.1-2.6

- **The Cartesian Coordinate System**

$x$  and  $y$  axis (label your scales on graphs)

Ordered pairs (points):  $(x, y)$

Origin:  $(0,0)$

- **Straight Lines**

$$\text{Slope} = m = \frac{y_2 - y_1}{x_2 - x_1}$$

Positive slope: line rises from lft to rt

Negative slope: line falls from lft to rt

Zero slope: Horizontal line

Undefined Slope: Vertical line

Equations of Lines

Pt-Slope Form:  $y - y_1 = m(x - x_1)$

Slope-Int Form:  $y = mx + b$

General Form:  $Ax + By + C = 0$

Horizontal line:  $y = a$

( $a$  is the  $y$ -value of every point on the line)

Vertical line:  $x = b$

( $b$  is the  $x$ -value of every point on the line)

Intercepts

$x$ -intercept = point where line crosses  $x$ -axis:

$(\#,0)$

$y$ -intercept = point where line crosses  $y$ -axis:

$(0,\#)$

Parallel Lines  $\implies m_1 = m_2$

(same slopes/diff.  $y$ -int)

- **1.3 - Linear Functions and Math. Models**

Functions

Domain

Range

Independent Variable

Dependent Variable

Linear Functions

Linear Depreciation:  $V(t) = mt + b$

$|m|$  = rate of depreciation

$b$  = value of asset at time zero

Scrap Value = lowest value asset attains

Linear Cost, Revenue and Profit

Cost:  $C(x) = cx + F$  where  $c$  is the cost to produce each unit and  $F$  is the fixed costs

Revenue:  $R(x) = sx$  where  $s$  is the selling price of each unit

Profit:  $P(x) = R(x) - C(x)$

Linear Supply and Demand

$S(x) = p = mx + b$

$D(x) = p = mx + b$

\*\*All points on supply and demand curves are of the form

$(x, p) = (\text{quantity, price})!!**$

- **1.4 - Intersection of Straight Lines**

Break-Even Point

$R(x) = C(x)$  (or  $P(x) = 0$  to find quantity)

$x$  = break-even quantity

$y$  = break-even revenue

Equilibrium Point

Supply = Demand

$x$  = equilibrium quantity

$y$  = equilibrium price

- **1.5 - The Method of Least Squares/Linear Regression**

Be able to use LinReg on your calculator to find the least-sq. line

Correlation coefficient ( $r$ ) - determines the amount of the data explained by the line

(Want  $|r|$  close to 1)

Be able to predict values, using the least-sq. line

- **2.1 - Systems of Linear Equations**

Two linear eqns  $\implies$  three cases

Unique Soln (intersecting lines  $\implies m_1 \neq m_2$ )

No Soln (parallel lines  $\implies$

$$m_1 = m_2 \text{ and } b_1 \neq b_2)$$

Infinitely Many Solns (same line  $\implies$

$$m_1 = m_2 \text{ and } b_1 = b_2)$$

General soln (parametric soln)

Specific solns

Setting up systems of equations

DEFINE YOUR VARIABLES!

- **2.2/2.3 - Solving System of Equations**

Gauss-Jordan (GJ) Elimination (Row Operations)

Interchange any two equations

Multiply an eqn by a non-zero constant

Add a multiple of one eqn to another

Reduced Row-Echelon Form

All zero rows must be below all non-zero rows

The first non-zero entry in each row is 1  
(leading 1)

In any two successive (non-zero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row

If a column contains a leading 1, then the other entries in that column are zeros

Solving a System

Put system in “nice” form (align variables/constants)

Place “nice” system in an augmented matrix

Use RREF on your calculator to reduce your system

Read system (eqns) from reduced system

Find soln

- **2.4 - Matrices**

Size (dimension):  $m \times n$  ( $m = \#$  rows,  $n = \#$  columns)

Matrix elements:  $a_{ij}$  (element in row  $i$  and col  $j$ )

Equality  $\implies$  all corresponding entries equal

Addition/Subtraction (matrices must be the same size)

Matrix Transpose ( $A^T$ ): switch rows and cols

Scalar Multiplication: multiply every entry by the scalar

- **2.5 - Multiplication of Matrices**

The  $\#$  of cols in the left matrix must equal the  $\#$  of rows in the right matrix.  
(Inner dimensions equal.)

If  $A$  is  $(m \times n)$  and  $B$  is  $(n \times p)$ , then  $AB$  is  $(m \times p)$ .  
(Outer dimensions give answer size.)

ORDER IS IMPORTANT!

Know how to multiply matrices by hand.

Identity Matrix: Square matrix ( $\#$  rows =  $\#$  cols) with 1's along the diagonal (from upper left to lower right) and 0's elsewhere

Be able to represent a system of eqns as a matrix eqn:  $AX = B$

Put system in “nice” form

$A$  = coefficient matrix

$X$  = variable matrix

$B$  = constant matrix

- **2.6 - Inverse of a Square Matrix**

Matrix must be square

Not all matrices have an inverse  
(“singular” means no inverse)

Inverse of  $A = A^{-1}$

$$AA^{-1} = A^{-1}A = I$$

A system in form  $AX = B \implies X = A^{-1}B$  if the inverse exists

Know how to solve a matrix equation