Department of Mathematics

O. G. Aberth, M. Aguiar, G. D. Allen, M. V. Anshelevich, W. Bangerth, D. L. Barrow, G. A. Battle, III, G. Berkolaiko, G. R. Blakley, H. P. Boas, A. Boggess (Head), A. Bonito, I. Borosh, L. Bowen, J. H. Bramble, G. Chen, C. K. T. Chui, A. Comech, P. Daripa, R. D. DeBlassie, R. A. DeVore, R. G. Douglas, K. J. Dykema, Y. Efendiev, T. Erdelyi, C. Foias, S. A. Fulling, S. C. Geller, R. Grigorchuk, J.-L. Guermond, R. A. Gustafson, D. J. Hartfiel, D. A. Hensley, J. M. Herrmann, A. M. Hobbs, P. Howard, W. B. Johnson, G. Kanschat, D. Kerr, T. R. Kiffe, P. Kuchment, H. E. Lacey, J. Landsberg, D. R. Larson, R. D. Lazarov, D. R. Lewis, P. Lima-Filho*, B. D. Lowe, L. Matusevich, C. J. Maxson, F. J. Narcowich, V. Nekrashevych, P. Nelson, Jr., D. Panchenko, G. Paovris, M. Papanikolas, J. E. Pasciak, C. Pearcy, W. L. Perry, G. Petrova, M. S. Pilant, G. Pisier, J. T. Pitts, A. Poltoratski, B. Popov, M. H. Rahe, K. R. Rajagopal, J. N. Reddy, C. Robles, J. M. Rojas, E. C. Rowell, W. Rundell, D. A. Sanchez, H. K. Schenck, J. F. Schielack, V. P. Schielack, Jr., T. Schlumprecht, N. Sivakumar, J. C. Slattery, K. C. Smith, R. R. Smith, F. Sottile, M. J. Stecher, P. F. Stiller, E. Straube, Z. Sunik, S. D. Taliaferro, P. Tretkoff, T. I. Vogel, Y. Vorobets, J. R. Walton, J. D. Ward, S. Witherspoon, C. H. Yan, P. B. Yasskin, M. P. Young, I. Zelenko, J. Zhou, J. Zinn

* Graduate Advisor

The Department of Mathematics offers graduate studies leading to the MS and PhD degrees in mathematics. Many of the course offerings are also suitable for graduate students pursuing degrees in engineering, science, geosciences, business, economics and education.

At the MS level, a student can pursue either a thesis or non-thesis degree. For the MS degree, a specialization in scientific computation, applied mathematics, financial mathematics or mathematics teaching is possible.

Satisfactory completion of the departmental qualifying exams is required of all students pursing a PhD In addition, the PhD degree requires a reading knowledge of Chinese, French, German, Russian or Spanish.

Admission to the Department's graduate programs is decided by the Graduate Programs Committee. Among the factors considered in admission decisions are: GRE General Test, undergraduate and graduate GPR, undergraduate academic background and achievement, letters of recommendation, GRE Subject Test in Mathematics (encouraged but not required).

Detailed information concerning programs and financial assistance may be obtained by writing the Graduate Programs Office, Department of Mathematics.

Mathematics (MATH)

601. Methods of Applied Mathematics I. (3-0). Credit 3. Methods of linear algebra, vector analysis and complex variables. Prerequisite: MATH 308 or equivalent.

602. Methods and Applications of Partial Differential Equations. (3-0). Credit 3. Classification of linear partial differential equations of the second order; Fourier series, orthogonal functions, applications to partial differential equations; special functions, Sturm-Liouville theory, application to boundary value problems; introduction to Green's functions; finite Fourier transforms. Prerequisites: MATH 601 or MATH 308 and 407.

603. Methods of Applied Mathematics II. (3-0). Credit 3. Tensor algebra and analysis; partial differential equations and boundary value problems; Laplace and Fourier transform methods for partial differential equations. Prerequisite: MATH 601 or 311.

604. Mathematical Foundations of Continuum Mechanics. (3-0). Credit 3. Mathematical description of continuum mechanics principles, including: tensor analysis, generalized description of kinematics and motion, conservation laws for mass and momentum; invariance and symmetry principles; application to generalized formulation of constitutive expressions for various fluids and solids. Prerequisites: MATH 410; MATH 451 or equivalent. Cross-listed with MEMA 604.

605. Mathematical Fluid Dynamics. (3-0). Credit 3. Derivation of basic equations of motion; Navier-Stokes equations; potential equations; some exact solutions in two and three dimensions; equations of boundary layer theory; vorticity-stream function formulation and vortex dynamics; introduction to hydrodynamic stability; introduction to equations of turbulence. Prerequisite: MATH 601 or equivalent.

606. Theory of Probability I. (3-0). Credit 3. Measure and integration, convergence concepts, random variables, independence and conditional expectation, laws of large numbers, central limit theorems, applications. Prerequisite: MATH 607 or approval of instructor.

607. Real Variables I. (3-0). Credit 3. Lebesgue measure and integration theory, differentiation, Lp-spaces, abstract integration, signed measures; Radon-Nikodym theorem, Riesz representation theorem, integration on product spaces. Prerequisite: MATH 447 or equivalent.

608. Real Variables II. (3-0). Credit 3. Banach spaces, theorems of Hahn-Banach and Banach-Steinhaus, the closed graph and open mapping theorems, Hilbert spaces, topological vector spaces and weak topologies. Prerequisite: MATH 607.

609. Numerical Analysis. (3-3). Credit 4. Interpolation, numerical evaluation of definite integrals and solution of ordinary differential equations; stability and convergence of methods and error estimates. Prerequisite: Knowledge of computer programming (C or FORTRAN).

610. Numerical Methods in Partial Differential Equations. (3-3). Credit 4. Introduction to finite difference and finite element methods for solving partial differential equations; stability and convergence of methods and error bounds. Prerequisite: MATH 417 or 609 or equivalent.

611. Introduction to Ordinary and Partial Differential Equations. (3-0). Credit 3. Basic theory of ordinary differential equations; existence and uniqueness, dependence on parameters, phase portraits, vector fields. Partial differential equations of first order, method of characteristics. Basic linear partial differential equations: Laplace equation, heat (diffusion) equation, wave equation and transport equation. Solution techniques and qualitative properties. Prerequisite: MATH 410 or equivalent or instructor's approval.

612. Partial Differential Equations. (3-0). Credit 3. Theory of linear partial differential equations; Sobolev spaces; elliptic equations (including boundary value problems and spectral theory); linear evolution equations of parabolic and hyperbolic types (including initial and boundary value problems). As time permits, additional topics might be included. Prerequisite: MATH 611 and MATH 607 or 641, or approval of instructor.

613. Graph Theory. (3-0). Credit 3. One or more broad areas of graph theory or network theory, such as planarity, connectivity, Hamiltonian graphs, colorings of graphs, automorphisms of graphs, or network theory. Prerequisite: MATH 431 or equivalent or approval of instructor.

614. Dynamical Systems and Chaos. (3-0). Credit 3. Discrete maps; continuous flows; dynamical systems; Poincaré maps; symbolic dynamics; chaos, strange attractors; fractals; computer simulation of dynamical systems. Prerequisites: MATH 308; MATH 601 or equivalent.

615. Introduction to Classical Analysis. (3-0). Credit 3. Set-theorectic preliminaries; Cantor-Schröder-Bernstein Theorem; review of sequences; limit inferior and limit superior; infinite products; metric spaces; convergence of functions; Dini's Theorem, Weierstrass Approximation Theorem; Monotone functions; bounded variation; Helly's Selection Theorem; Riemann-Stieltjes integration; Fourier series; Fejer's Theorem; Parseval's Identify; Bernstein's Theorem on absolutely convergent Fourier series. Prerequisite: Math 409 or equivalent.

617. Theory of Functions of a Complex Variable I. (3-0). Credit 3. Holomorphic functions, complex integral theorems, Runge's theorem, residue theorem, Laurent series, conformal mapping, harmonic functions. Prerequisite: MATH 410.

618. Theory of Functions of a Complex Variable II. (3-0). Credit 3. Infinite products, Weierstrass factorization theorem, Mittag-Leffler's theorem, normal families, Riemann mapping theorem, analytic continuation, Picard's theorems and selected topics. Prerequisite: MATH 617.

619. Applied Probability. (3-0). Credit 3. Measure Theory; Lebesgue integration; random variables; expectation; condition expectation martingales and random walks; designed for beginning graduate students in mathematics, statistics, the sciences and engineering and students in economics and finance with a strong mathematical background. Prerequisites: MATH 409 and 411.

622. Differential Geometry I. (3-0). Credit 3. Surfaces in 3-D space and generalizations to submanifolds of Euclidean space; smooth manifolds and mappings; tensors; differential forms; Lie groups and algebras; Stokes' theorem; deRham cohomology; Frobenius theorem; Riemannian manifolds. Prerequisites: MATH 304 or equivalent; approval of instructor.

623. Differential Geometry II. (3-0). Credit 3. Curvature of Riemannian manifolds; vector bundles; connections; Maurer-Cartan Form; Laplacian; geodesics; Chern-Gauss-Bonnet theorem; additional topics to be selected by the instructor. Prerequisites: MATH 622 or approval of instructor.

625. Applied Stochastic Differential Equations. (3-0). Credit 3. Stochastic integration, Ito Calculus and applications of stochastic differential equations to finance and engineering. Prerequisite: MATH 619.

626. **Analytic Number Theory. (3-0). Credit 3.** Analytic properties of the Riemann zeta function and Dirichlet L-functions; Dirichlet characters; prime number theorem; distribution of primes in arithmetic progressions; Siegel's theorem; the large sieve inequalities; Bombieri-Vinogradov theorem. Prerequisite: Math 617.

627. **Algebraic Number Theory. (3-0). Credit 3.** Algebraic number fields and rings of algebraic integers; arithmetic in algebraic number fields; ideals; unique factorization of ideals; ideal classes and the class group; finiteness of the class number; Minkowski's theorem; Dirichlet's unit theorem; quadratic and cyclotomic number fields; splitting of primes in extension fields. Prerequisite: MATH 653 or approval of instructor.

628. **Mathematics of Finance. (3-0). Credit 3.** Pricing of financial derivatives in different market models; discrete models Arrow-Debreu, Binomial model, Hedging; Stochastic calculus; Brownian Motion, stochastic integrals, Ito formula; continuous model, Black-Scholes formula for pricing European and American options; equivalent Martingale Measures, pricing or exotic options. Prerequisite: MATH 606 or 619 or approval of instructor.

629. **History of Mathematics. (3-0). Credit 3.** Major events in the evolution of mathematical thought from ancient times to the present, the development of various important branches of mathematics, including numeration, geometry, algebra, analysis, number theory, probability, and applied mathematics. Prerequisite: MATH 304 or equivalent.

630. **Combinatorics. (3-0). Credit 3.** This is an introduction at the graduate level to the fundamental ideas and results of combinatorics, including enumerative techniques, sieve methods, partially ordered sets and generating functions. Prerequisite: undergraduate discrete math course or permission of instructor.

636. **Topology I. (3-0). Credit 3.** Set theory, topological spaces, generalized convergence, compactness, metrization, connectedness, uniform spaces, function spaces. Prerequisite: Approval of instructor.

637. **Topology II. (3-0). Credit 3.** Continuation of MATH 636. Prerequisite: MATH 636 or approval of instructor.

639. **Iterative Techniques. (3-3). Credit 4.** Numerical methods for solving linear and nonlinear equations and systems of equations; eigenvalue problems. Prerequisites: Elementary linear algebra and knowledge of computer programming (C or FORTRAN).

640. **Linear Algebra for Applications. (3-0). Credit 3.** Review of linear algebra; spectral theory in inner product spaces; decomposition theorems; duality theory and multilinear algebra; tensor products; applications. May be taken concurrently with MATH 641. Prerequisite: MATH 304 or equivalent.

641. **Analysis for Applications I. (3-0). Credit 3.** Review of preliminary concepts; sequence and function spaces; normed linear spaces, inner product spaces; spectral theory for compact operators; fixed point theorems; applications to integral equations and the calculus of variations. Prerequisites: MATH 447 and 640 or approval of instructor.

642. **Analysis for Applications II. (3-0). Credit 3.** Distributions and differential operators; transform theory; spectral theory for unbounded self-adjoint operators; applications to partial differential equations; asymptotics and perturbation theory. Prerequisite: MATH 641.

643. **Algebraic Topology I. (3-0). Credit 3.** Fundamental ideas of algebraic topology, homotopy and fundamental group, covering spaces, polyhedra. Prerequisite: Approval of instructor.

644. **Algebraic Topology II. (3-0). Credit 3.** Homology and cohomology theory. Prerequisite: MATH 643.

645. **A Survey of Mathematical Problems I. (3-0). Credit 3.** A survey of problems in various branches of mathematics, such as logic, probability, graph theory, number theory, algebra and geometry. Prerequisites: MATH 409, 415, 423 or approval of instructor.

646. **A Survey of Mathematical Problems II. (3-0). Credit 3.** A survey of problems in various branches of mathematics such as algebra, geometry, differential equations, real analysis, complex analysis, calculus of variations. Prerequisite: MATH 645 or approval of instructor.

647. **Mathematical Modelling. (3-0). Credit 3.** The process and techniques of mathematical modelling; covers a variety of applications areas and models such as ordinary and partical differential equations, stochastic models, discrete models and problems involving optimization. Prerequisite: MATH 442 or approval of instructor.

648. **Computational Algebraic Geometry. (3-0). Credit 3.** Broad introduction to algorithmic algebraic geometry, including numerical and complexity theoretic aspects; theory behind the most efficient modern algorithms for polynomial system solving and the best current quantitative/geometric estimates on algebraic sets over various rings is derived. Prerequisite: MATH 653 or approval of instructor.

650. Several Complex Variables. (3-0). Credit 3. Introduction to function theory in several complex variables with an emphasis on the analytic and partial differential equations aspects of the subject. Prerequisites: MATH 608 and 618 or equivalents.

651. Optimization I. (3-0). Credit 3. Fundamentals of mathematical analysis underlying theory of constrained optimizations for a finite number of variables, necessary and sufficient conditions for constrained extrema of equality constraint problems, sufficient conditions for fulfillment of constraint qualification, computational methods for concave programming problems and applications. Prerequisite: MATH 410 or approval of instructor.

652. Optimization II. (3-0). Credit 3. Necessary conditions of calculus of variations, elementary theory of games, formulation of basic control problem, Hestenes' necessary conditions for optimal control, transformations, methods of computation and applications. Prerequisite: MATH 651.

653. Algebra I. (3-0). Credit 3. Survey of groups, rings, ideals. Prerequisite: MATH 415 or approval of instructor.

654. Algebra II. (3-0). Credit 3. Survey of modules, field extensions, Galois theory. Prerequisite: MATH 653 or approval of instructor.

655. Functional Analysis I. (3-0). Credit 3. Normed linear spaces, duality theory, reflexivity, operator theory. Banach algebras, spectral theory, representation theory. Prerequisite: MATH 608.

656. Functional Analysis II. (3-0). Credit 3. Topological linear spaces, locally convex spaces, duality in locally convex spaces, ordered topological vector spaces, distribution theory, applications to analysis. Prerequisite: MATH 655.

657. Spline Analysis and Applications. (3-0). Credit 3. Review of fundamental concepts of approximation, polynomials and other tools; basic univariate spline theory including bases, computational algorithms and approximation power; Bezier curves; applications to interpolation, discrete approximation, data fitting; computer-aided geometric design (CAGD), nonlinear rational B-splines (NURBS). Prerequisite: MATH 304 or equivalent.

658. Applied Harmonic Analysis. (3-0). Credit 3. Fourier series and Fourier Transform; discrete (fast) Fourier transform; discrete cosine transform; local cosine transform; Radon transform; filters; harmonic analysis on the sphere; radial, periodic and spherical basis functions; applications. Prerequisites: MATH 304; MATH 308 or equivalent.

660. Computational Linear Algebra. (3-0). Credit 3. Techniques in matrix computation: elimination methods, matrix decomposition, generalized inverses, orthogonalization and least-squares, eigenvalue problems and singular value decomposition, iterative methods and error analysis. Prerequisite: MATH 417 or equivalent or CSCE 442 or equivalent. Cross-listed with CSCE 660.

661. Mathematical Theory of Finite Element Methods. (3-0). Credit 3. Will develop basic mathematical theory of finite element method; construction of finite element spaces and piece-wise polynomial approximation; Ritz-Galerkin methods and variational crimes; energy and maximum norm estimates; mixed finite element method; applications to diffusion-reaction problems.

662. Seminar in Algebra. (3-0). Credit 3. Problems, methods and recent developments in algebra. May be taken five times for credit as content varies. Prerequisite: Approval of instructor.

663. Seminar in Analysis. (3-0). Credit 3. Problems, methods and recent developments in analysis. May be taken five times for credit as content varies. Prerequisite: Approval of instructor.

664. Seminar in Applied Mathematics. (3-0). Credit 3. Problems, methods and recent developments in applied mathematics. This course may be taken five times for credit as content varies. Prerequisite: Approval of instructor.

666. Seminar in Geometry. (3-0). Credit 3. Problems, methods and recent developments in geometry. This course may be taken five times for credit as content varies. Prerequisite: Approval of instructor.

667. Foundations and Methods of Approximation. (3-0). Credit 3. Existence, uniqueness and characterization of best approximations; polynomial and rational approximants; Bernstein polynomials; Bernstein and Markov inequalities; ridge functions; approximation from shift-invariant subspaces; orthogonal polynomials; neural networks, radial basis functions, scattered-data surface fitting; subdivision analysis. Prerequisites: MATH 407 and 409.

668. **Wavelet Analysis. (3-0). Credit 3.** Time-frequency analysis, integral wavelet transform, multiresolutional analysis, dyadic wavelets and inversions, frames, classification of wavelets, dual basis and a duality principle, wavelet decompositions and reconstructions, spline-wavelets, zero-crossings of spline-wavelet series, wavelet packets, multivariate wavelets. Prerequisites: MATH 304, 409, 417 or equivalents.

669. **Seminar in Mathematical Biology. (3-0). Credit 3.** Problems, methods and recent developments in Mathematical Biology. Prerequisite: Approval of instructor.

670. **Applied Mathematics I. (3-0). Credit 3.** Mathematical tools of applied mathematics; Fredholm alternative; integral operators; Green's functions; unbounded operators; Stone's theorem; distributions; convolutions; Fourier transforms; applications. Prerequisite: MATH 642 or equivalent.

671. **Applied Mathematics II. (3-0). Credit 3.** Mathematical tools of applied mathematics; Sobolev spaces; convexity; variational inequalities; variational methods for partial differential equations; maximum principles; elements of nonlinear analysis; compact operators; fixed point theorems; applications. Prerequisite: MATH 670 or equivalent.

672. **Hydrodynamic Stability. (3-0). Credit 3.** Instability mechanisms; instability of interfacial and free surface flows; thermal instability, centrifugal instability, instability of inviscid and viscous parallel shear flows; fundamental concepts and applications of nonlinear instability; the onset of turbulence; various transitions to turbulence. Prerequisites: MATH 601 or equivalent; MATH 605 or equivalent.

673. **Information, Secrecy and Authentication I. (3-0). Credit 3.** Preliminaries; probability, information, entropy, signals, channels: group-theoretic view of messages: contemporary secrecy and digital signature systems; one-time pads, DES, RSA, DSS, wheels, LFSR-based systems; analog scramblers; key exchange, key management, secret sharing, access structures; measures of security. Prerequisites: Graduate classification and approval of instructor. Cross-listed with CSCE 673.

674. **Information, Secrecy and Authentication II. (3-0). Credit 3.** Classical and recent attacks: login, compression, error control and genetic codes; finite and infinite codes; matrices, graphs, duals, groups, morphisms, composites, products, rates and classification of codes; the confusion/diffusion/arithmetic/calculus extension of Shannon's two design primitives. Prerequisites: MATH 673; graduate classification or approval of instructor. Cross-listed with CSCE 674.

676. **Finite Element Methods in Scientific Computing. (3-0). Credit 3.** Basic finite element methods; structure of finite element codes; assembling linear systems of equations and algorithmic aspects; linear iterative solvers; adaptive mesh refinement; vector-valued and mixed problems; nonlinear problems; visualization; parallelization aspects. Additional topics may be chosen by instructor. Prerequisites: MATH 610, ENGR finite element class or MATH 419/609 plus instructor approval. Knowledge of C++.

684. **Professional Internship. Credit 1 to 6.** Directed internship in an organization to provide students with professional experience in organization settings appropriate to the student's career objectives. Prerequisite: Approval of department head.

685. **Directed Studies. Credit 1 to 6 each semester.** Offered to enable students to undertake and complete, with credit, limited investigations not within their thesis research and not covered by any other courses in the curriculum. Prerequisite: Approval of instructor.

689. **Special Topics in… Credit 1 to 4.** Selected topics in an identified area of mathematics. May be repeated for credit. Prerequisite: Approval of instructor.

691. **Research. Credit 1 or more each semester.** Research for thesis or dissertation.

694. **Mathematical Laboratory. (0-2). Credit 1.** Generic computing or problem-solving laboratory. May be taken multiple times for credit. Taken concurrently with a lecture course for which it will serve as the laboratory section. Prerequisite: Graduate classification.

695. **Frontiers in Mathematical Research. (3-0). Credit 3.** This course is designed to acquaint the graduate student with the present status of investigative work in a variety of mathematical fields. Content will depend on the availability of visiting lecturers who will be selected because of distinguished international recognition in their fields of research. May be taken two times for credit. Prerequisite: Graduate classification.

696. **Mathematical Communication and Technology. (3-0). Credit 3.** Techniques of oral, written and electronic communication of mathematics; effective classroom and seminar presentation; TEX, AMS-TEX, and LATEX, hypertext; Internet application; Maple and Matlab; classroom use of computer graphics. Prerequisite: Approval of instructor.