MATH 151 (816-818)
Final Examination
December 12, 2007

Name: _____

ID#: _____

The exam consists of 19 questions, the first 15 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 15 mark your answers on the ScanTron form.

1. [4 pts] Given $\mathbf{a} = \langle -1, 2 \rangle$ and $\mathbf{b} = \langle 5, -3 \rangle$, find $3\mathbf{a} + \mathbf{b}$.

   (a) $\langle -8, 9 \rangle$

   (b) $\langle 4, -1 \rangle$

   (c) $\langle -3, 6 \rangle$

   (d) $\langle 2, 3 \rangle$

   (e) $\langle 5, 0 \rangle$

2. [4 pts] Given the function $f(x) = (\ln(x^4 + 1))^2$, find $f'(1)$.

   (a) 4

   (b) 0

   (c) 1

   (d) $\ln 4$

   (e) $\ln 16$

3. [4 pts] Find $\int e^{2x}(1+e^{2x})^3\,dx$.

(a) $e^{8x}+C$

(b) $e^{2x}(1+e^{2x})^4+C$

(c) $\frac{1}{8}e^{2x}(1+e^{2x})^4+C$

(d) $\frac{1}{8}(1+e^{2x})^4+C$

(e) $\frac{1}{4}(1+e^{2x})^4+C$

4. [4 pts] Find an equation for the line tangent to the curve $\mathbf{r}(t)=\langle \sin t, t^2+t+1\rangle$ at the point corresponding to $\mathbf{r}(0)$.

(a) $y=x+1$

(b) $y=1$

(c) $x=0$

(d) $y=2x+3$

(e) $y=3x-1$

5. [4 pts] Use Newton's method to find a second approximation x_2 to a root of the equation $5x^2 - e^x = 0$ given the initial approximation $x_1 = 1$.

(a) $\dfrac{5}{10 - e}$

(b) 1

(c) $5 - 2e$

(d) $\frac{5}{9}$

(e) $\dfrac{5 - e}{10 - e}$

6. [4 pts] Find the value of c which makes the function

$$f(x) = \begin{cases} \dfrac{\sin(cx)}{e^{-x} - 1} & \text{if } x < 0 \\ 2 + \displaystyle\int_0^x \tan^{-1}(e^t)\, dt & \text{if } x \geq 0 \end{cases}$$

continuous everywhere.

(a) -2

(b) -1

(c) 0

(d) 1

(e) 2

7. [4 pts] Which of the following is true?

(a) $\displaystyle\int_0^1 \sqrt{1+x^4}\,dx < 1$

(b) $\displaystyle\int_0^1 -e^{-x^3}\,dx \geq 0$

(c) $\displaystyle\int_1^2 \tan^{-1}(x^2)\,dx \geq \pi$

(d) $\displaystyle\int_{-1}^1 \frac{\sin x}{x^8+3}\,dx = 0$

(e) $\displaystyle\int_0^1 x^4\,dx = \frac{1}{4}$

8. [4 pts] Find the derivative of the function $f(x) = \tan^{-1}(e^{x^2})$.

(a) $\dfrac{2xe^{x^2}}{1+e^{2x^2}}$

(b) $\dfrac{2xe^{x^2}}{1+e^{x^4}}$

(c) $\dfrac{4x}{1+e^{2x^2}}$

(d) $\dfrac{2x}{1+e^{2x^2}}$

(e) $\dfrac{2xe^{2x^2}}{1+e^{2x^2}}$

9. [4 pts] A horizontal force of 20 N is applied to move a box up a ramp that is 10 m long and inclined at an angle of 30 degrees to the horizontal. What is the work done on the box?

   (a) 200 J
   (b) $100\sqrt{3}$ J
   (c) 0 J
   (d) 100 J
   (e) 20 J

10. [4 pts] In which of the following intervals does the equation $x^4 + 8x - 1 = 0$ have a solution?

   (a) $(-2, -1)$
   (b) $(-1, 0)$
   (c) $(0, 1)$
   (d) $(1, 2)$
   (e) $(2, 3)$

11. [4 pts] Let y be defined implicitly in terms of x by the equation $x^2e^y - y^2 = 1$. Find y' when $(x, y) = (1, 0)$.

   (a) 0
   (b) $-\frac{1}{2}$
   (c) -1
   (d) -2
   (e) e

12. [4 pts] Find the derivative of the function $f(x) = \dfrac{(x+4)^{24}(x-3)^8}{(x+3)^{57}(x+6)^5}$.

(a) $\dfrac{24(x+4)^{23}(x-3)^8 + 8(x+4)^{24}(x-3)^7}{(x+3)^{56}(x+6)^4}$

(b) $\dfrac{24(x+4)^{23}(x-3)^8 + 8(x+4)^{24}(x-3)^7}{(x+3)^{56}(x+6)^4} f(x)$

(c) $\left(\dfrac{24}{x+4} + \dfrac{8}{x-3} - \dfrac{57}{x+3} - \dfrac{5}{x+6}\right) f(x)$

(d) $\left(\dfrac{1}{24(x+4)} + \dfrac{1}{8(x-3)} - \dfrac{1}{57(x+3)} - \dfrac{1}{5(x+6)}\right) f(x)$

(e) $\dfrac{24(x+4)^{23}(x-3)^8 + 8(x+4)^{24}(x-3)^7}{(x+3)^{114}(x+6)^{10}}$

13. [4 pts] Find the linear approximation to the function $f(x) = (x+2)e^{x-1}$ at the point $x = 1$.

(a) $L(x) = 2ex - 2e + 3$

(b) $L(x) = 3x - 2$

(c) $L(x) = 3x + 3$

(d) $L(x) = 4ex - e$

(e) $L(x) = 4x - 1$

14. [4 pts] Suppose that $f'(x) = x^2 - 4x + 15$ on the interval $(1, 3)$. What can be said about f?

(a) f has a local minimum at some point in $(1, 3)$

(b) f has a local maximum at some point in $(1, 3)$

(c) the graph of f is concave upward on $(1, 3)$

(d) the graph of f is concave downward on $(1, 3)$

(e) the graph of f has a point of inflection $(c, f(c))$ for some c in $(1, 3)$

15. [4 pts] Given the function $f(x) = \displaystyle\int_x^{2x} \frac{1}{t^4 + t^2 + 1}\, dt$, find $f'(0)$.

(a) $\frac{1}{3}$

(b) $-\frac{1}{3}$

(c) 0

(d) 1

(e) 2

16. [10 pts] Compute $\displaystyle\int_0^3 \left(2x + \frac{3x}{\sqrt{x+1}}\right) dx.$

17. [10 pts] Find an equation for the tangent line to the graph of the function $f(x) = 2\sin^2 x$ at $x = \frac{\pi}{4}$.

18. [10 pts] Two cars start moving away from an intersection. Car A travels due north and its distance from the intersection after t seconds is $(t+1)^t - 1$ meters. Car B travels due south and its speed after t seconds is $\frac{2t}{t^2+1}$ meters per second. How fast is the distance between the two cars increasing when $t = 1$?

19. [10 pts] Find the largest possible area of a rectangle whose base lies on the x-axis and whose two vertices above the x-axis lie on the curve $y = 80 - x^4$.