

MATH 220.903  
Examination 1  
Solutions

1. (a) Statement. (b) Open sentence. (c) Open sentence. (d) Statement. (e) Open sentence.
2. (a) Example: For every even integer  $n$ ,  $n^3$  is even.  
(b) Example: For every set  $A$ ,  $\emptyset \subseteq A$ .
3. (a) Example: For all sets  $A$  and  $B$ ,  $A \subseteq B$ .  
(b) Example: For every set  $A$ ,  $\emptyset \notin \mathbf{P}(A)$ .
4. (a) (i) True. (ii) Every real-valued function on  $[0, 1]$  is bounded.  
(b) (i) True. (ii) There exists a partition  $\mathcal{P}$  of  $\{1, 2, \dots, 10\}$  such that  $|\mathcal{P}| = 2$  and  $|A| < 5$  for each  $A \in \mathcal{P}$ .
5. (a) If  $A \cup B = B$  then  $A \subseteq \emptyset$ .  
(b) If  $A \cup B \neq B$  then  $A \not\subseteq \emptyset$ .
6. Suppose that  $n$  is a multiple of 3. Then we can write  $n = 3k$  for some integer  $k$ . We then have
$$n^3 + n^2 + 3 = (3t)^3 + (3t)^2 + 3 = 3(9t^3 + 3t^2 + 1).$$
Since  $9t^3 + 3t^2 + 1$  is an integer, we conclude that  $n^3 + n^2 + 3$  is a multiple of 3.
7. (a)  $\{0\}$ .  
(b)  $\mathbb{R}$ .  
(c)  $(-\infty, -\frac{1}{2}]$ .
8. (a) 3. (b) 2. (c) 1. (d) 5.