MATH 251
Examination 1
February 21, 2007

Name: _____

ID#: _____

The exam consists of 11 questions, the first 8 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 8 circle the correct answer.

1. [5] The cosine of the angle between the two vectors $\langle 1, 2, 2\rangle$ and $\langle 3, -4, 0\rangle$ is

   (a) $-\frac{1}{45}$
   (b) 15
   (c) -5
   (d) $-\frac{1}{3}$
   (e) 0

2. [5] The range of the function $f : \mathbb{R}^2 \to \mathbb{R}$ given by $f(x, y) = e^{x+y}$ is

   (a) $\mathbb{R}$
   (b) $\{z \in \mathbb{R} : z > 0\}$
   (c) $\{(x, y) \in \mathbb{R}^2 : x + y > 0\}$
   (d) $\{z \in \mathbb{R} : z \geq 1\}$
   (e) $\mathbb{R}^2$

3. [5] How many two-dimensional unit vectors $\mathbf{u}$ satisfy the equation $\mathbf{u} \cdot \langle 2, 5 \rangle = 0$?

(a) 0
(b) 1
(c) 2
(d) 3
(e) infinitely many

4. [5] Identify the quadric surface $z = y^2 - 3x^2$.

(a) elliptic paraboloid
(b) hyperbolic paraboloid
(c) ellipsoid
(d) parabolic cylinder
(e) hyperboloid of one sheet

5. [5] The directional derivative of the function $f(x, y) = 2xy^2$ at the point $(1, 2)$ in the direction of the vector $\mathbf{v} = \langle 2, 1 \rangle$ is

(a) 24
(b) 2
(c) -24
(d) $\frac{2}{\sqrt{3}}$
(e) $\frac{24}{\sqrt{5}}$

6. [5] Find $\frac{\partial w}{\partial s}$ where $w = xyz$, $x = st^2$, $y = e^t$, and $z = 3s + t$.

(a) $(6st^2 + t^3)e^t$.
(b) $t^2 + 3$
(c) $(t^2 + 3)e^t$
(d) $5st^2 + 3s + t$
(e) $-s + t$

7. [5] Which of the following is a normal vector for the plane $x + 3y - 2z = 7$?

(a) $\langle 3, -2, 7\rangle$
(b) $\langle 1, -1, 1\rangle$
(c) $\langle 0, 0, 7\rangle$
(d) $\langle -2, 3, 1\rangle$
(e) $\langle 2, 6, -4\rangle$

8. [5] Which of the following vectors satisfies the equation $\mathbf{a} \times \langle 0, 0, 1\rangle = 0$?

(a) $\mathbf{a} = \langle 5, 0, 0\rangle$
(b) $\mathbf{a} = \langle 0, 5, 0\rangle$
(c) $\mathbf{a} = \langle 0, 0, 5\rangle$
(d) $\mathbf{a} = \langle 1, 1, 1\rangle$
(e) $\mathbf{a} = \langle 1, 1, 0\rangle$

9. [20] Consider the two planes $x - y + z = 1$ and $x - z = 2$.

   (a) Find symmetric equations for their line of intersection.

   (b) Give parametric equations for a line which intersects neither plane.

10. [20] Use differentials to approximate the value of the function $f(x, y, z) = x^2yz^3$ at the point $(0.99, 2.99, 1.01)$.

11. [20] Show that $\lim_{(x,y)\to(1,0)} \dfrac{x^2-2x-y^2+1}{x^2-2x+y^2+1}$ doesn't exist.