MATH 251
Examination 2
March 28, 2007

Name: ______________________________

ID#: ______________________________

The exam consists of 11 questions, the first 8 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 8 circle the correct answer.

1. [5] Which of the following pairs (r, θ) represents the Cartesian point $(1, 1)$ in polar coordinates?

   (a) $(\sqrt{2}, \pi)$
   (b) $(\sqrt{2}, \frac{\pi}{4})$
   (c) $(1, \pi)$
   (d) $(1, \frac{5\pi}{4})$

2. [5] Consider the function $f(x, y) = x^2 - y^2 + 2y$. Which of the following is true?

   (a) f has a local minimum at $(0, 1)$.
   (b) f has a local maximum at $(0, 1)$.
   (c) $(0, 1)$ is a saddle point for f.
   (d) $(0, 1)$ is not a critical point for f.

3. [5] How many critical points does the function $f(x, y) = e^{x+y}$ have?

(a) none
(b) one
(c) two
(d) infinitely many

4. [5] The iterated integral $\int_0^1 \int_0^1 2x \, dy \, dx$ is equal to

(a) -2
(b) 0
(c) 1
(d) 4

5. [5] For every continuous function f on the disk $D = \{(x, y) : x^2 + y^2 \leq 1\}$ the double integral $\iint_D f(x, y) \, dA$ can be evaluated as

(a) $\displaystyle\int_0^{2\pi} \int_0^1 f(r\cos\theta, r\sin\theta) r \, dr \, d\theta$

(b) $\displaystyle\int_0^{2\pi} \int_0^1 f(r\cos\theta, r\sin\theta) \, dr \, d\theta$

(c) $\displaystyle\int_{-1}^{1} \int_0^{\sqrt{1-x^2}} f(x, y) \, dy \, dx$

(d) $\displaystyle\int_0^{1} \int_{-\sqrt{1-y^2}}^{\sqrt{1-y^2}} f(x, y) \, dx \, dy$

6. [5] Let D be the disk $\{(x, y) : x^2 + y^2 \leq 1\}$. Which of the following is false?

(a) $\iint_D e^x \, dA \geq 0$.
(b) Every continuous function on D attains an absolute maximum value on D.
(c) The boundary of D is described by the polar equation $r = \sin 2\theta$.
(d) Every continuous function on D is integrable on D.

7. [5] Let D be the region $\{(x, y) : 0 \leq x \leq 2,\ 0 \leq y \leq x^2\}$. Then $\iint_D e^{xy} \, dA$ is equal to

(a) $\displaystyle\int_0^2 \int_0^4 e^{xy} \, dx \, dy$

(b) $\displaystyle\int_0^2 \int_0^4 e^{xy} \, dy \, dx$

(c) $\displaystyle\int_0^4 \int_0^{\sqrt{y}} e^{xy} \, dx \, dy$

(d) $\displaystyle\int_0^2 \int_0^{x^2} e^{xy} \, dy \, dx$

8. [5] The graph of the polar equation $r = 2\cos\theta$ is a circle of radius

(a) $\frac{1}{2}$
(b) 1
(c) 2
(d) 4

9. [20] Find the absolute maximum and minimum values of the function $f(x, y) = e^{x^2-4x+y^2-2y}$ on the closed disk of radius two centered at $(2, 0)$.

10. [20] Let D be the region in $\mathbb{R}^2$ which is bounded by the upper half of the circle $x^2 + y^2 = 1$, the curve with polar equation $r = \theta + 1$ for $0 \leq \theta \leq \pi$, and the portion of the x-axis between $x = -\pi - 1$ and $x = -1$. Let E be the volume in $\mathbb{R}^3$ which lies between the xy-plane and the paraboloid $z = x^2 + y^2$ and above the region D.

(a) Calculate $\displaystyle\iint_D 12\sqrt{x^2 + y^2}\, dA$.

(b) Use part (a) to calculate $\displaystyle\iiint_E \frac{36}{\sqrt{x^2 + y^2}}\, dV$.

11. [20] Consider a lamina which occupies the region in the plane bounded by the lines $x = 0$, $y = 0$, and $y = 1 - x$. Suppose the lamina has density function $\rho(x, y) = 24x$. Find the center of mass of the lamina.