

MATH 251.505
Final Examination
May 7, 2007

Name: _____

ID#: _____

The exam consists of 13 questions, the first 8 of which are multiple choice. The point value for a question is written next to the question number. There is a total of 100 points. No aids are permitted.

For questions 1 to 8 circle the correct answer.

1. [5] Consider the vector field $\mathbf{F}(x, y, z) = z\mathbf{i} + y\mathbf{j} + x\mathbf{k}$. Then $\text{div } \mathbf{F}$ is equal to

- (a) 0
- (b) 1
- (c) 3
- (d) $x + y + z$
- (e) $\frac{1}{2}y^2$

2. [5] Consider the vector field $\mathbf{F}(x, y, z) = xy\mathbf{j} + z\mathbf{k}$. Then the third component of $\text{curl } \mathbf{F}$ is equal to

- (a) y
- (b) $x + 1$
- (c) 0
- (d) $xy + z$
- (e) xyz

3. [5] The vector field $\mathbf{F}(x, y, z) = y \sin z \mathbf{i} + x \sin z \mathbf{j} + xy \cos z \mathbf{k}$ is the gradient vector field of which of the following functions?

- (a) $f(x, y, z) = xy \cos z$
- (b) $f(x, y, z) = xy(\sin z + \cos z)$
- (c) $f(x, y, z) = -xy \sin x$
- (d) $f(x, y, z) = 38 + xy \sin z$
- (e) $f(x, y, z) = (1 + xy) \sin z$

4. [5] For which value of c is the vector field $\mathbf{F}(x, y, z) = yz^2 \mathbf{i} + xz^2 \mathbf{j} + cxyz \mathbf{k}$ conservative?

- (a) -2
- (b) -1
- (c) 0
- (d) 1
- (e) 2

5. [5] Let $f : \mathbb{R}^3 \rightarrow \mathbb{R}$ be a differentiable function such that $f(x, y, z) > 0$ for all $(x, y, z) \in \mathbb{R}^3$. Which of the following is false?

- (a) $\iint_S f(x, y, z) dS \geq 0$ for every smooth surface S in \mathbb{R}^3 .
- (b) $\int_C f(x, y, z) ds > 0$ for every smooth curve C in \mathbb{R}^3 .
- (c) For smooth curves C in \mathbb{R}^3 , the line integral $\int_C f(x, y, z) ds$ depends only on the endpoints of C .
- (d) $\int_C \nabla f \cdot d\mathbf{r} = 0$ for every simple closed smooth curve C in \mathbb{R}^3 .
- (e) For smooth curves C in \mathbb{R}^3 , the line integral $\int_C \nabla f \cdot d\mathbf{r}$ depends only on the endpoints of C .

6. [5] Evaluate the surface integral $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where $\mathbf{F}(x, y, z) = z\mathbf{i} - yz\mathbf{j} + x\mathbf{k}$ and S is the part of the graph of the function $g(x, y) = ye^x$ that lies over the region in the xy -plane bounded by the parabola $y = x^2$ and the lines $y = 0$ and $x = 1$.

- (a) $\frac{1}{3}$
- (b) $\frac{1}{4}$
- (c) 0
- (d) 1
- (e) e

7. [5] Consider the curve C defined parametrically by $\mathbf{r}(t) = t\mathbf{i} + t^3\mathbf{j} + t^2\mathbf{k}$ where $0 \leq t \leq 1$. Then $\int_C z \, dy$ is equal to

- (a) $\frac{1}{3}$
- (b) $\frac{1}{2}$
- (c) 1
- (d) 0
- (e) $\frac{3}{5}$

8. [5] The area of the part of the graph of the function $f(x, y, z) = x^2 + y^2 + 10$ that lies above the triangle D in the xy -plane with vertices $(0, 0)$, $(1, 0)$, and $(1, 1)$ is equal to

- (a) $\iint_D (x^2 + y^2 + 10)\sqrt{1 + 4x^2 + 4y^2} \, dA$
- (b) $\int_0^1 \int_0^x \sqrt{1 + 4x^2 + 4y^2} \, dy \, dx$
- (c) $\iint_D (x^2 + y^2 + 10) \, dA$
- (d) $\iint_D dA$
- (e) $\int_0^1 \int_0^{\sqrt{1-x^2}} (x^2 + y^2 + 10) \, dy \, dx$

9. [12] Find $\text{curl } \mathbf{F}$ and $\text{div } \mathbf{F}$.

(a) $\mathbf{F}(x, y, z) = x \sin y \mathbf{i} + e^y \mathbf{j} + z^4 \mathbf{k}$.

(b) $\mathbf{F} = \text{curl } \mathbf{G}$ where $\mathbf{G}(x, y, z) = x^2 \mathbf{i} + xyz \mathbf{j} + x^2 z \mathbf{k}$.

10. [12] Let C be a smooth curve in \mathbb{R}^3 with initial point $(0, 0, 0)$ and terminal point $(-3, 0, 8)$. Use the Fundamental Theorem for Line Integrals to evaluate $\int_C \mathbf{F} \cdot d\mathbf{r}$ where $\mathbf{F}(x, y, z) = yze^x \mathbf{i} + (ze^x + z^2) \mathbf{j} + (ye^x + 2yz + 1) \mathbf{k}$.

11. [12] Use the Divergence Theorem to evaluate $\iint_S \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = x^2y \mathbf{i} + \frac{xz^3}{4 + z^6} \mathbf{j} + (-2xy + 15\sqrt{x^2 + y^2})z \mathbf{k}$$

and S is the surface of the solid consisting of all points $(x, y, z) \in \mathbb{R}^3$ such that $0 \leq z \leq 1 - x^2 - y^2$, with outward orientation.

12. [12] Use Green's Theorem to evaluate $\int_C y^2 e^x dx + (4xy^2 + 2ye^x) dy$ where C is the circle $x^2 + y^2 = 4$ with clockwise orientation.

13. [12] Use Stokes' Theorem to evaluate $\iint_S \text{curl } \mathbf{F} \cdot d\mathbf{S}$ where

$$\mathbf{F}(x, y, z) = ze^{x \sin y} \mathbf{i} + (x^2 + y^2 + z^3) \mathbf{j} + \sin(xz) \mathbf{k}$$

and S is the graph of the function $g(x, y) = x^4 y^3 (1 - x^2 - y^2)$ with domain $\{(x, y) \in \mathbb{R}^2 : x^2 + y^2 \leq 1 \text{ and } y \geq 0\}$, with upward orientation.