MATH 308
Assignment #1 Solutions

Section 1.1

2. This is a linear second-order ODE with independent variable x and dependent variable y.
8. This is a nonlinear second-order ODE with independent variable x and dependent variable y.

Section 1.2

20. (a) Note that $\phi'(x) = me^{mx}$ and $\phi''(x) = m^2e^{mx}$. Substituting into the given DE then yields, after simplication and factoring, $(m+5)(m+1)e^{mx} = 0$, which holds for all x precisely when $m = -5$ or -1.

    (b) Note that $\phi'(x) = me^{mx}$, $\phi''(x) = m^2e^{mx}$, and $\phi'''(x) = m^3e^{mx}$. Substituting into the given DE then yields, after simplication and factoring, $m(m+1)(m+2)e^{mx} = 0$, which holds for all x precisely when $m = 0$, -1, or -2.

28. We can write the DE as $\frac{dx}{dt} = f(t,x)$ where $f(t,x) = \sin t - \cos x$. Then $\frac{\partial f}{\partial x} = \sin x$. Since f and $\frac{\partial f}{\partial x}$ are continuous everywhere, by Theorem 1 the given initial value problem has a unique solution on some interval containing π.

Section 1.3

6. (a) To find the slope of the solution curve through $(1, \pi/2)$ we substitute these values into the right-hand side of the DE to get $1 + \sin(\pi/2)$, which is equal to 2.

   (b) Since $\sin y \geq -1$ for all values of y, when $x > 1$ we have $x + \sin y > 0$. So the right-hand side of the DE is greater than zero when $x > 1$, which means that every solution is increasing for $x > 1$.

   (c) Implicitly differentiating the DE with respect to x, substituting $x + \sin y$ for $\frac{dy}{dx}$, and using the double angle formula $\sin 2y = 2 \sin y \cos y$, we get

$$\frac{d^2y}{dx^2} = 1 + (\cos y)\frac{dy}{dx} = 1 + (\cos y)(x + \sin y) = 1 + x \cos y + \frac{1}{2} \sin 2y.$$

   (d) Substituting $(0,0)$ for (x,y) in both the original DE and the equation derived in part (c), we get $\frac{dy}{dx}(0) = 0$ and $\frac{d^2y}{dx^2}(0) = 1 > 0$. By the second derivative test, the solution curve has a local minimum at $(0,0)$.