

Fall 2006  
Week-in-Review #10  
*courtesy: Kendra Kilmer*  
(covering Sections 8.4, 8.5, and 8.6)

### Section 8.4

- A **binomial experiment** has the following properties:
    - The total number of trials is fixed in advance.
    - Each trial has two outcomes: "success" and "failure".
    - The trials are independent of each other.
    - The probability of success is the same for each trial.
  - The random variable  $X$  which represents the number of successes in a binomial experiment is known as a **Binomial Random Variable**.
  - If we are dealing with a binomial random variable, we must determine the number of trials ( $n$ ), the probability of success in each trial ( $p$ ), and the number of successes desired ( $r$ ). We can then compute:
    - $P(X = r) = C(n, r)p^r(1 - p)^{(n-r)}$
    - $E(X) = np$
    - $Var(X) = np(1 - p)$
    - $\sigma = \sqrt{np(1 - p)}$
1. A fair six-sided die is rolled 50 times.
    - (a) What is the probability of rolling a five 10 times?
    - (b) What is the probability of rolling an even number 20 times?
    - (c) What is the probability of rolling a 3 at most 10 times?
    - (d) What is the probability of rolling an odd number at least 35 times?
    - (e) How many times would you expect to roll an even number?
    - (f) What is the standard deviation of the number of times you roll an even number?
  2. On a given class day 15% of the students are not listening in their class. If 25 students are randomly selected,
    - (a) What is the probability that exactly 10 of the students are listening?
    - (b) What is the probability that at least 5 of the students are listening?
    - (c) What is the probability that between 5 and 15 students, inclusive, are listening?
    - (d) How many students would you expect to listening in the group of 25?
    - (e) What is the standard deviation of the number of students listening in this group of 25?
  3. Suppose that one-fifth of the restaurants in town are in violation of the health code. If an inspector inspects eight of the restaurants,
    - (a) What is the probability that the first two restaurants will pass the inspection and the remaining six will fail the inspection?
    - (b) What is the probability that just two restaurants will pass the inspection?

### Section 8.5

- If  $X$  is a **continuous random variable**, a **probability density function** is defined to represent the probability distribution of  $X$ . The curve lies completely above the  $x$ -axis and the total area under this curve is 1.

- A normal random variable is defined by its mean ( $\mu$ ) and standard deviation ( $\sigma$ ). The probability density function associated with a normal random variable has its peak directly above the mean and is symmetric about a vertical line passing through the mean. The **standard normal variable**  $Z$  has a mean of 0 and a standard deviation of 1.
- To find probabilities associated with a Normal Random Variable, press 2nd VARS and the select option 2:normalcdf. On your homescreen, enter normalcdf(left bound, right bound, mean, standard deviation).
- If you are given a probability and asked to find a bound use 3:invNorm. If you are trying to find  $a$ , then  $a = \text{invNorm}(\text{total area under the curve to the left of } a, \text{ mean, standard deviation})$ .

4. Let  $Z$  be the standard normal variable. Find the following:
  - (a)  $P(Z \geq 1.5)$
  - (b)  $P(Z < 2)$
  - (c)  $P(-1.5 < Z \leq 2)$
5. Let  $X$  be a normal random variable with  $\mu = 80$  and  $\sigma = 5$ . Find the following:
  - (a)  $P(X \leq 70)$
  - (b)  $P(X > 75)$
  - (c)  $P(45 \leq X \leq 90)$
6. Let  $Z$  be the standard normal variable. In each of the following, find  $z$ .
  - (a)  $P(Z \leq z) = 0.7524$
  - (b)  $P(Z \geq z) = 0.4268$
  - (c)  $P(Z \geq -z) = 0.2657$
  - (d)  $P(-z \leq Z \leq z) = 0.7587$

### Section 8.6

- Use normalcdf and invNorm in word problems when you know the random variable has a normal distribution.
  - When using a normal distribution to approximate a binomial probability, you need to
    - Identify  $n$  and  $p$ .
    - Compute  $\mu$  and  $\sigma$ .
    - Draw a rough sketch of the histogram to determine the area under the curve you are wanting to compute. Don't forget to add or subtract 0.5.
    - Use normalcdf to find the desired area under the normal curve.
7. Annual salaries at a local company are normally distributed with a mean of \$30,000 and a standard deviation of \$5,000. If a person is randomly selected from the company, what is the probability that their annual salary is more than \$25,000?
  8. Grades on a history exam were normally distributed with a mean of 75 and a standard deviation of 10. If 8% of the students received an A, what is the lowest score a student may have and still obtain an A?
  9. A fair coin is flipped 1000 times. Use the appropriate normal distribution to approximate the binomial distribution.
    - (a) What is the probability of the coin landing on heads at least 520 times.
    - (b) What is the probability of the coin landing on heads between 500 and 600 times, inclusive.
    - (c) What is the probability of the coin landing on heads less than 400 times?
  10. A basketball player has a 75% chance of making a free throw. What is the probability that he makes more than 200 shots if he attempts 500 free throws? (Use the appropriate normal distribution to approximate the binomial distribution.)