

Fall 2006
Week-in-Review #10
courtesy: Kendra Kilmer
(covering Sections 8.4, 8.5, and 8.6)

Section 8.4

- A **binomial experiment** has the following properties:
 - The total number of trials is fixed in advance.
 - Each trial has two outcomes: "success" and "failure".
 - The trials are independent of each other.
 - The probability of success is the same for each trial.
- The random variable X which represents the number of successes in a binomial experiment is known as a **Binomial Random Variable**.
- If we are dealing with a binomial random variable, we must determine the number of trials (n), the probability of success in each trial (p), and the number of successes desired (r). We can then compute:
 - $P(X = r) = C(n, r)p^r(1 - p)^{(n-r)}$
 - $E(X) = np$
 - $Var(X) = np(1 - p)$
 - $\sigma = \sqrt{np(1 - p)}$

1. A fair six-sided die is rolled 50 times.

(a) What is the probability of rolling a five 10 times?

(b) What is the probability of rolling an even number 20 times?

(c) What is the probability of rolling a 3 at most 10 times?

(d) What is the probability of rolling an odd number at least 35 times?

(e) How many times would you expect to roll an even number?

(f) What is the standard deviation of the number of times you roll an even number?

2. On a given class day 15% of the students are not listening in their class. If 25 students are randomly selected,
- (a) What is the probability that exactly 10 of the students are listening?

 - (b) What is the probability that at least 5 of the students are listening?

 - (c) What is the probability that between 5 and 15 students, inclusive, are listening?

 - (d) How many students would you expect to listening in the group of 25?

 - (e) What is the standard deviation of the number of students listening in this group of 25?
3. Suppose that one-fifth of the restaurants in town are in violation of the health code. If an inspector inspects eight of the restaurants,
- (a) What is the probability that the first two restaurants will pass the inspection and the remaining six will fail the inspection?

 - (b) What is the probability that just two restaurants will pass the inspection?

Section 8.5

- If X is a **continuous random variable**, a **probability density function** is defined to represent the probability distribution of X . The curve lies completely above the x -axis and the total area under this curve is 1.
- A normal random variable is defined by its mean (μ) and standard deviation (σ). The probability density function associated with a normal random variable has its peak directly above the mean and is symmetric about a vertical line passing through the mean. The **standard normal variable** Z has a mean of 0 and a standard deviation of 1.
- To find probabilities associated with a Normal Random Variable, press 2nd VARS and the select option 2:normalcdf. On your homescreen, enter normalcdf(left bound, right bound, mean, standard deviation).
- If you are given a probability and asked to find a bound use 3:invNorm. If you are trying to find a , then $a = \text{invNorm}(\text{total area under the curve to the left of } a, \text{ mean, standard deviation})$.

4. Let Z be the standard normal variable. Find the following:

(a) $P(Z \geq 1.5)$

(b) $P(Z < 2)$

(c) $P(-1.5 < Z \leq 2)$

5. Let X be a normal random variable with $\mu = 80$ and $\sigma = 5$. Find the following:

(a) $P(X \leq 70)$

(b) $P(X > 75)$

(c) $P(45 \leq X \leq 90)$

6. Let Z be the standard normal variable. In each of the following, find z .

(a) $P(Z \leq z) = 0.7524$

(b) $P(Z \geq z) = 0.4268$

(c) $P(Z \geq -z) = 0.2657$

(d) $P(-z \leq Z \leq z) = 0.7587$

Section 8.6

- Use `normalcdf` and `invNorm` in word problems when you know the random variable has a normal distribution.
 - When using a normal distribution to approximate a binomial probability, you need to
 - Identify n and p .
 - Compute μ and σ .
 - Draw a rough sketch of the histogram to determine the area under the curve you are wanting to compute. Don't forget to add or subtract 0.5.
 - Use `normalcdf` to find the desired area under the normal curve.
7. Annual salaries at a local company are normally distributed with a mean of \$30,000 and a standard deviation of \$5,000. If a person is randomly selected from the company, what is the probability that their annual salary is more than \$25,000?

8. Grades on a history exam were normally distributed with a mean of 75 and a standard deviation of 10. If 8% of the students received an A, what is the lowest score a student may have and still obtain an A?

9. A fair coin is flipped 1000 times. Use the appropriate normal distribution to approximate the binomial distribution.

(a) What is the probability of the coin landing on heads at least 520 times.

(b) What is the probability of the coin landing on heads between 500 and 600 times, inclusive.

(c) What is the probability of the coin landing on heads less than 400 times?

10. A basketball player has a 75% chance of making a free throw. What is the probability that he makes more than 200 shots if he attempts 500 free throws? (Use the appropriate normal distribution to approximate the binomial distribution.)