

Fall 2006  
Week-in-Review #2  
*courtesy: Kendra Kilmer*  
(covering Sections 2.1-2.4)

**Section 2.1**

- When solving a system of linear equations, our solution will be in one of the following forms:
    - Unique Solution
    - No Solution
    - Infinitely Many Solutions
  - When setting up a word problem, always make sure to define your variables first!
1. Solve the following systems of linear equations using any method.
- (a)
- $$\begin{aligned}2x - y &= 1 \\5x + y &= 27\end{aligned}$$
- (b)
- $$\begin{aligned}-2x + 3y &= 21 \\4x - 6y &= 12\end{aligned}$$
- (c)
- $$\begin{aligned}x - 5y &= 15 \\-3x + 15y &= -45\end{aligned}$$
2. Jack and Jill decide to go into the business of making table decorations for weddings. They will be decorating three different sizes of tables. A small table seats 4 people, a medium table seats 8 people, and a large table seats 12 people. For a certain wedding of 344 people, the bride has decided her table decorating budget is \$2,520 and she wants twice as many medium tables as small and large tables combined. If it cost \$40 to decorate a small table, \$60 to decorate a medium table, \$75 to decorate a large table, how many tables of each size should they use? Assume that the entire budget is used and there will be no extra seats. (Set-up but do not solve.)
3. A theater has a seating capacity of 1200 and charges \$3 for children, \$4 for students, and \$5 for adults. At a certain screening with full attendance, there were half as many adults as children and students combined. The receipts totaled \$5100. How many children, students, and adults attended the show? (Set-up but do not solve.)
4. I want to make three different types of cakes. The recipe to make 1 pineapple cake calls for 1 cup of flour, 2 cups of sugar, and 3 cups of water. The recipe to make 1 chocolate cake calls for 2 cups of flour, 1 cup of sugar, and 3 cups of water. The recipe to make 1 lemon cake calls for 1 cup of flour, 3 cups of sugar, and 1 cup of water. If I have 50 cups of flour, 80 cups of sugar, and 100 cups of water, how many cakes of each should I make if I want to use all of my ingredients? (Set-up but do not solve.)

**Sections 2.2, 2.3**

- The goal of the **Gauss-Jordan Elimination Method** is to get the augmented matrix in **Row Reduced Form**. A matrix is in **Row Reduced Form** when:
  - (a) Each row of the coefficient matrix consisting entirely of zeros lies below any other row having nonzero entries.
  - (b) The first nonzero entry in each row is 1 (called a leading 1)
  - (c) In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
  - (d) If a column contains a leading 1, then the other entries in that column are zeros.
- Note:** We only consider the coefficient side (left side) of the augmented matrix when determining whether the matrix is in row-reduced form.
- To put a matrix in **Row Reduced Form**, there are three valid **Row Operations**:
  - (a) Interchange any two rows ( $R_i \leftrightarrow R_j$ )
  - (b) Replace any row by a nonzero constant multiple of itself ( $cR_i$ )
  - (c) Replace any row by the sum of that row and a constant multiple of any other row ( $R_i + cR_j$ ).

5. Determine whether each of the following matrices is in Row-Reduced Form.

(a)  $\left[ \begin{array}{ccc|c} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{array} \right]$

(b)  $\left[ \begin{array}{cccc|c} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array} \right]$

(c)  $\left[ \begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 4 & 2 \end{array} \right]$

6. Solve the system of linear equations using Gauss-Jordan Elimination.

$$9y = -3x - 15$$

$$2x = -8y + 6$$

7. Pivot the given system about the boxed element.

$$\left[ \begin{array}{ccc|c} 1 & -3 & -4 & 17 \\ 0 & \boxed{7} & 2 & -8 \\ 0 & 4 & 5 & 9 \end{array} \right]$$

8. Find the solution(s) to the following systems of linear equations:

(a) Solve the system we set-up in problem 2.

(b)

$$6z - 3x = 3y + 9$$

$$2x - y + 3z = 7$$

$$x = 2y - 5z$$

(c)

$$2x + 2y - z = 7$$

$$2x - y - 3z = 3$$

$$3y + 2z = 4$$

(d)

$$2x_1 - x_2 + 5x_3 + 2x_4 = 2$$

$$2x_1 - x_2 + 3x_3 + 4x_4 = -2$$

$$x_1 - 2x_2 + 5x_3 - 3x_4 = 4$$

## Section 2.4

- A matrix is an ordered rectangular array of numbers. A matrix with  $m$  rows and  $n$  columns has dimensions  $m \times n$ .
- Our basic matrix operations are addition, subtraction, scalar multiplication, and transpose.

9. Given  $A = \begin{bmatrix} -2 & 8 & 2 & 11 \\ 3 & -7 & 5 & 6 \\ 8 & 2 & -5 & 3 \end{bmatrix}$

What are the dimensions of  $A$ ? Find  $a_{34}$ ,  $a_{12}$ , and  $a_{23}$ .

10. Given  $A = \begin{bmatrix} 2 & 1 \\ -5 & 7 \\ 8 & f \end{bmatrix}$   $B = \begin{bmatrix} 1 & 9 & -7 \\ 4 & k & 8 \end{bmatrix}$

Find  $C$  where  $C = 3A + B^T$ .