Fall 2006

Week-in-Review #2

courtesy: Kendra Kilmer

(covering Sections 2.1-2.4)

Section 2.1

- When solving a system of linear equations, our solution will be in one of the following forms:
  - Unique Solution
  - No Solution
  - Infinitely Many Solutions
- When setting up a word problem, always make sure to define your variables first!

1. Solve the following systems of linear equations using any method.

   (a)
   $$2x - y = 1$$
   $$5x + y = 27$$

   (b)
   $$-2x + 3y = 21$$
   $$4x - 6y = 12$$

   (c)
   $$x - 5y = 15$$
   $$-3x + 15y = -45$$

2. Jack and Jill decide to go into the business of making table decorations for weddings. They will be decorating three different sizes of tables. A small table seats 4 people, a medium table seats 8 people, and a large table seats 12 people. For a certain wedding of 344 people, the bride has decided her table decorating budget is $2,520 and she wants twice as many medium tables as small and large tables combined. If it cost $40 to decorate a small table, $60 to decorate a medium table, $75 to decorate a large table, how many tables of each size should they use? Assume that the entire budget is used and there will be no extra seats. (Set-up but do not solve.)

3. A theater has a seating capacity of 1200 and charges $3 for children, $4 for students, and $5 for adults. At a certain screening with full atendance, there were half as many adults as children and students combined. The receipts totaled $5100. How many children, students, and adults attended the show? (Set-up but do not solve.)

4. I want to make three different types of cakes. The recipe to make 1 pineapple cake calls for 1 cup of flour, 2 cups of sugar, and 3 cups of water. The recipe to make 1 chocolate cake calls for 2 cups of flour, 1 cup of sugar, and 3 cups of water. The recipe to make 1 lemon cake calls for 1 cup of flour, 3 cups of sugar, and 1 cup of water. If I have 50 cups of flour, 80 cups of sugar, and 100 cups of water, how many cakes of each should I make if I want to use all of my ingredients? (Set-up but do not solve.)

Sections 2.2, 2.3

- The goal of the **Gauss-Jordan Elimination Method** is to get the augmented matrix in **Row Reduced Form**. A matrix is in **Row Reduced Form** when:
  (a) Each row of the coefficient matrix consisting entirely of zeros lies below any other row having nonzero entries.
  (b) The first nonzero entry in each row is 1 (called a leading 1)
  (c) In any two successive (nonzero) rows, the leading 1 in the lower row lies to the right of the leading 1 in the upper row.
  (d) If a column contains a leading 1, then the other entries in that column are zeros.

  Note: We only consider the coefficient side (left side) of the augmented matrix when determining whether the matrix is in row-reduced form.
- To put a matrix in **Row Reduced Form**, there are three valid **Row Operations**:
  (a) Interchange any two rows ($R_i \leftrightarrow R_j$)
  (b) Replace any row by a nonzero constant multiple of itself (cR_i)
  (c) Replace any row by the sum of that row and a constant multiple of any other row ($R_i + cR_j$).

5. Determine whether each of the following matrices is in Row-Reduced Form.

(a) $\left[\begin{array}{cc|c} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 0 \end{array}\right]$

(b) $\left[\begin{array}{cccc|c} 1 & 3 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 \end{array}\right]$

(c) $\left[\begin{array}{ccc|c} 1 & 3 & 0 & 0 \\ 0 & 0 & 1 & 3 \\ 0 & 0 & 4 & 2 \end{array}\right]$

6. Solve the system of linear equations using Gauss-Jordan Elimination.

$$9y = -3x - 15$$

$$2x = -8y + 6$$

7. Pivot the given system about the boxed element.

$$\left[\begin{array}{ccc|c} 1 & -3 & -4 & 17 \\ 0 & \boxed{7} & 2 & -8 \\ 0 & 4 & 5 & 9 \end{array}\right]$$

8. Find the solution(s) to the following systems of linear equations:

   (a) Solve the system we set-up in problem 2.

   (b)

$$6z - 3x = 3y + 9$$
$$2x - y + 3z = 7$$
$$x = 2y - 5z$$

(c)

$$\begin{aligned} 2x + 2y - z &= 7 \\ 2x - y - 3z &= 3 \\ 3y + 2z &= 4 \end{aligned}$$

(d)

$$\begin{aligned} 2x_1 - x_2 + 5x_3 + 2x_4 &= 2 \\ 2x_1 - x_2 + 3x_3 + 4x_4 &= -2 \\ x_1 - 2x_2 + 5x_3 - 3x_4 &= 4 \end{aligned}$$

Section 2.4

- A matrix is an ordered rectangular array of numbers. A matrix with m rows and n columns has dimensions $m \times n$.
- Our basic matrix operations are addition, subtraction, scalar multiplication, and transpose.

9. Given A= $\begin{bmatrix} -2 & 8 & 2 & 11 \\ 3 & -7 & 5 & 6 \\ 8 & 2 & -5 & 3 \end{bmatrix}$

   What are the dimensions of A? Find a_{34}, a_{12}, and a_{23}.

10. Given A= $\begin{bmatrix} 2 & 1 \\ -5 & 7 \\ 8 & f \end{bmatrix}$ B= $\begin{bmatrix} 1 & 9 & -7 \\ 4 & k & 8 \end{bmatrix}$

    Find C where $C = 3A + B^T$.