Fall 2006

Week-in-Review #4

courtesy: Kendra Kilmer

(covering Sections 3.1-3.3 and 5.1)

(most problems from *Finite Mathematics* by Waner/Costenoble)

Section 3.1

- To graph a system of linear inequalities:(The instructions below use reverse shading. Please note that some instructors might have taught you to shade the true region.)
  - Solve all inequalities for y.
  - Graph the corresponding equations. If the inequality is a strict inequality ($<$ or $>$) draw the line as a dotted line to represent that the points on the line are NOT part of the solution. If the inequaility is inclusive ($\leq$ or $\geq$) draw the line as a solid line to represent that the points on the line are part of the solution.
  - Determine which half-plane satisfies the inequality and then cross-out(shade) the points we don't want. You can do this by using a test point or by using the following "rules". Please note that these "rules" only work once we have the inequality solved for y:
    - If you have $y \leq f(x)$ or $y < f(x)$ the true region lies below the line. Thus, we cross-out(shade) the points above the line.
    - If you have $y \geq f(x)$ or $y > f(x)$ the true region lies above the line. Thus, we cross-out(shade) the points below the line.
  - Our **solution set (feasible region)** consists of all points left unshaded.
- A solution set is **bounded** if all of the points in our solution set can be enclosed by a circle. Otherwise, we say that the solution set is **unbounded**.
- A corner point is a point along the boundary of our feasible region in which we have a sharp turn.

1. Sketch the region that corresponds to the given inequalities, say whether the region is bounded or unbounded, and find the coordinates of all corner points.

   (a) $x + y \geq 5$
   $x \leq 10$
   $y \leq 8$
   $x \geq 0, y \geq 0$

   (b) $20x + 10y \geq 100$
   $10x + 20y \geq 100$
   $10x + 10y \geq 80$
   $x \geq 0, y \geq 0$

Section 3.2

- A Linear Programming Problem consists of a linear objective function to be maximized or minimized subject to constraints in the form of linear equations or inequalities.
- When setting up a linear programming problem, make sure to precisely define your variables, include your objective (maximize or minimize) and objective function along with all of your constraints.

2. Each serving of Gerber Mixed Cereal for Baby contains 60 calories and 11 grams of carbohydrates. Each serving of Gerber Mango Tropical Fruit Dessert contains 80 calories and 21 grams of carbohydrates. If the cereal costs 30 cents per serving and the dessert costs 50 cents per serving, and you want to provide your child with at least 140 calories and at least 32 grams of carbohydrates, how can you do so at the least cost? (Set-up but do not solve the Linear Programming Problem.)

3. In February 2002, each episode of "Becker" was typically seen in 8.3 million homes, while each episode of "The Simpsons" was seen in 7.5 million homes. Your marketing services firm has been hired to promote Bald no More, Inc.'s hair replacement process by buying at least 30 commercial spots during episodes of "Becker" and "The Simpsons." The cable company running "Becker" has quoted a price of $2000 per spot, while the cable company showing "The Simpsons" has quoted a price of $1500 per spot. Bald No More's advertising budget for TV commercials is $70,000, and it would like no more than 50% of the total number of spots to appear on "The Simpsons." How many spots should you purchase on each show to reach the most homes? (Set-up but do not solve the Linear Programming Problem.)

Section 3.3

- If a linear programming problem has a solution, we are guaranteed that it will occur at a corner point of the feasible region.
- If the feasible region is bounded then the objective function has both a maximum and minimum value.
- If the feasible region is unbounded and the coefficients of the variables in the objective function are both nonnegative, then the objective function has only a minimum value provided that $x \geq 0$ and $y \geq 0$ are two of the constraints.
- If the feasible region is the empty set, the linear programming problem has no solution.

  Method of Corners for Bounded Solution Set
  - Graph the feasible region.
  - Find the coordinates of all corner points.
  - Make a table and evaluate the objective function at all corner points to see which yields the optimal solution. (Note: If the objective function is optimized at two corner points, then the objecitve function is optimized at every point along the line segment connecting these two points as well. We say we have infinitely many solutions.)
- Please note that if we have an unbounded feasible region, depending on the coefficients in our objective function, a maximum or minimum value may not be possible.

4. Maximize $P = 3x + 2y$
Subject to $0.2x + 0.1y \leq 1$
$0.15x + 0.3y \leq 1.5$
$10x + 10y \leq 60$
$x \geq 0, y \geq 0$

5. Maximize $P = 2x + 3y$
Subject to $0.1x + 0.2y \geq 1$
$2x + y \geq 10$
$x \geq 0, y \geq 0$

6. Minimize $C = 2x + 4y$
Subject to $0.1x + 0.1y \geq 1$
$x + 2y \geq 14$
$x \geq 0, y \geq 0$

7. You manage an ice cream factory that makes two flavors: Creamy Vanilla and Continental Mocha. Into each quart of Creamy Vanilla go 2 eggs and 3 cups of cream. Into each quart of Continental Mocha go 1 egg and 3 cups of cream. You have in stock 500 eggs and 900 cups of cream. You make a profit of $3 on each quart of Creamy Vanilla and $4 on each quart of Continental Mocha.
   (a) How many quarts of each flavor should you make in order to earn the largest profit?
   (b) Are there any leftover resources? Be specific.

Section 5.1

- **Simple Interest:** $A = P(1 + rt)$ where A is the accumulated amount, P is the principle amount, r is the annual interest rate as a decimal, and t is time in years.
- **Compound Interest:** $A = P\left(1 + \frac{r}{m}\right)^{mt}$ where A, P, r, and t represent the same as above and m is the number of compounding periods per year.
- **Continuous Compound Interest:** $A = Pe^{rt}$ where A, P, r, and t represent the same as above and $e \approx 2.718281828....$
- **TVM Solver:**

  N =the total number of compounding periods

  $I\%$ = interest rate (as a percentage)

  PV = present value (principal amount). Entered as a negative number if invested, a positive number if borrowed.

  PMT = payment amount (0 if no payments are involved)

  FV =future value (accummulated amount)

  $P/Y = C/Y$=the number of compounding periods per year.

  Move the cursor to the value you are solving for and hit ALPHA and then ENTER. In all of the problems we do make sure that END is highlighted at the bottom of the screen. This represents that payments are received at the end of each period.
- The **Effective Rate of Interest** gives the equivalent interest rate if compounding was only done once a year. Use option C:Eff(on the finance menu. Enter as Eff(*interest rate as a percentage,number of compounding periods per year*)

8. How much would you have to deposit in an account earning 4.5% simple interest if you wanted to have $1000 after 6 years?
9. How much would you have in 8 years if you invest $1,500 into an account earning 5.4% compounded quarterly?
10. How long will it take an investment to double if it is earning 3.5% annual interest compounded monthly?
11. What is the future value of $35,000 invested at 6.5% annual interest compounded continuously for 5 years?
12. Which would give you a better return: 3.89% compounded daily or 3.91% compounded quarterly?