Fall 2006

Week-in-Review #5

courtesy: Kendra Kilmer

(covering Sections 5.2, 5.3, 6.1, and 6.2)

Sections 5.2 and 5.3

- **Annuity:** A sequence of payments made at regular time intervals.
- In this course, we will study annuities with the following properties:
  - The terms are given by fixed time intervals.
  - The periodic payments are equal in size.
  - The payments are made at the end of the payment periods.
  - The payment periods coincide with the interest conversion periods.
- **TVM Solver:**
  - N =the total number of compounding periods
  - $I\%$ = interest rate (as a percentage)
  - PV = present value (principal amount). Entered as a negative number if invested, a positive number if borrowed.
  - PMT = payment amount (0 if no payments are involved)
  - FV =future value (accummulated amount)
  - $P/Y = C/Y$=the number of compounding periods per year.
  - Move the cursor to the value you are solving for and hit ALPHA and then ENTER. In all of the problems we do make sure that END is highlighted at the bottom of the screen. This represents that payments are received at the end of each period.

1. If quarterly deposits of \$30 are put into an account earning 3.5% per year compounded quarterly, how much will be in the account at the end of 30 years?

2. Nick currently has \$500 in his travel funds account that earns interest at 2.45% per year compounded monthly. He plans on depositing \$75 each month to save for a trip to Europe. If he needs \$5,000 for the trip to Europe, when will he be able to go?

3. Meghan deposits \$10 every week into an account in which the interest is compounded weekly. In 3 years she discovers that her investment has grown to \$2,000. What is the effective rate of interest earned on this account?

4. In 1996, Lisa and Kevin bought a home for \$125,000. They put 5% down and then financed the remaining price of the home with a 30 year mortgage at an 8.54% annual interest rate compounded monthly on the unpaid balance.
   (a) What were their monthly payments?
   (b) In 2006 they decided to refinance the home with a 20 year mortgage at a 6.5% annual interest rate compounded monthly on the unpaid balance. What are their new monthly payments?
   (c) How much money are they saving in interest by refinancing?

5. Chris has been living it up and has accrued \$6,000 of credit card debt on a card that charges 18.4% per year compounded monthly on the unpaid balance. But lucky for Chris the minimum payment due each month is only \$95.
   (a) Assuming he doesn't make any more purchases, how long will it take Chris to pay off his debt?
   (b) If Chris decides to pay \$50 extra a month, how long will it take Chris to pay off his debt?
   (c) How much money is he saving by paying a little extra each month?

Section 6.1

- We can either use **roster notation** or **set-builder notation** to represent a set.
- Two sets A and B are **equal**, written $A = B$, if and only if they have exactly the same elements.
- If every element of a set A is also an element of a set B, then we say that A is a **subset** of B and write $A \subseteq B$.
- If A and B are sets such that $A \subseteq B$ but $A \neq B$, then we say A is a **proper subset** of B written $A \subset B$
- The set that contains no elements is called the **empty set** and is denoted by $\emptyset$. It is a subset of all sets.
- The **universal set** is the set of all elements of interest in a particular problem.
- We use **Venn Diagrams** to visually represent sets. The universal set U is represented by a rectangle and subsets of U are represented by circles inside of the rectangle.
- The **union** of A and B, written $A \cup B$ is the set of all elements that belong to either A or B or both.
- The set of elements in common with the sets A and B, written $A \cap B$, is called the **intersection** of A and B.
- The **complement** of A, denoted A^c is the set of all elements in U that are not in A.
- Two sets A and B are **disjoint** if $A \cap B = \emptyset$.
- **De Morgan's Laws:** Let A and B be sets. Then $(A \cup B)^c = A^c \cap B^c$ and $(A \cap B)^c = A^c \cup B^c$.

6. Let $U = \{1, 2, 3, 4, 5, 6, 7, 8, 9, 10\}$, $A = \{1, 3, 5, 7, 9\}$, $B = \{2, 4, 6, 8, 10\}$, $C = \{1, 5, 8, 9\}$, $D = \{8, 10\}$. Find the following:
   (a) $A \cup C$
   (b) $B \cap D$
   (c) C^c
   (d) $A \cup (B \cap D)$
   (e) $C \cap (A \cup D)^c$

7. Using the sets from the previous example, determine whether each statement is true or false:
   (a) $D \subset B$
   (b) $\{2, 4, 6, 8, 10\} \subset B$

(c) $8 \in D$

(d) $\{1, 8\} \in C$

8. List all subsets of the set A where $A = \{x | x \text{ is an integer between 3 and 5 inclusive}\}$.

9. For each part, draw a Venn Diagram with 3 sets, A, B, and C, and shade the region that represents each set.

   (a) $B \cap A^c \cap C^c$

   (b) $C \cap (A \cup B)$

   (c) $A \cup (B \cap C)^c$

Section 6.2

- $n(A)$ represents the number of elements in a set.
- $n(A \cup B) = n(A) + n(B) - n(A \cap B)$
- We can label each region in a Venn Diagram with the number of elements in it to sort out the given information.

10. If $n(A) = 15$, $n(B) = 23$, and $n(A \cap B) = 5$, what is $n(A \cup B)$?

11. A Universal Set consists of 3 subsets. We are given the following information:

    - $n(A \cap B^c \cap C^c) = 10$
    - $n(A) = 25$
    - $n(B \cap C) = 10$
    - $n(C) = 30$
    - $n(B^c) = 35$
    - $n(A \cap C \cap B^c) = 9$
    - $n(A \cap B \cap C) = 2$
    - $n(C^c) = 22$.

    Fill out the Venn Diagram with this information and use it to find $n(B \cup C)$.

12. A survey was conducted of 100 student's dining preferences. They were all asked their preference on eating at Chipotle, Fitzwilly's, and Potbelly's. The following information was determined:

    - 20 students like only Fitzwilly's.
    - 30 students like exactly two of the restaurants.
    - 50 students like Potbelly's
    - 5 students like all three restaurants.
    - 15 students like Chipotle and Fitzwilly's.
    - 8 students like only Fitzwilly's and Potbelly's
    - 60 students do not like Chipotle.

    Fill out the Venn Diagram with this information and use it to determine how many students do not like Fitzwilly's.