Fall 2006

Week-in-Review #8

courtesy: Kendra Kilmer

(covering Sections 7.5, 7.6, and 8.1)

(problems from *Finite Mathematics* by Waner/Costenoble and *Finite Mathematics* by Young/Lee/Long/Gradening)

Sections 7.5 and 7.6

- If A and B are events in an experiment and $P(A) \neq 0$, then the **conditional probability** that the event B will occur given that the event A has already occurred is $P(B|A) = \dfrac{P(A \cap B)}{P(A)}$
- **Product Rule:** $P(A \cap B) = P(A) \cdot P(B|A)$
- Tree Diagrams
- If A and B are **independent events** then $P(A|B) = P(A)$ and $P(B|A) = P(B)$. Futhermore, two events are independent if and only if $P(A \cap B) = P(A) \cdot P(B)$
- Bayes' Theorem

1. The following table lists the number of passengers who were on the *Titanic* and the number of passengers who survived, according to class of ticket.

	Children		**Women**		**Men**		**Totals**	
	On	Survived	On	Survived	On	Survived	On	Survived
First Class	6	6	144	140	175	57	325	203
Second Class	24	24	165	76	168	14	357	114
Third Class	79	27	93	80	462	75	634	182
Total	109	57	402	296	805	146	1316	499

(a) What is the probability that a randomly selected passenger was second class?

(b) What is the overall probability of surviving?

(c) What is the probability of a first-class passenger surviving?

(d) What is the probability of a child who was also in the third class surviving?

(e) Given that the survivor is from first class, what is the probability that she was a woman?

(f) Given that a male has survived, what is the probability that he was in third class?

2. If two fair dice are rolled, what is the probability that the sum is 8 given that the sum is greater than 7?

3. A bicycle factory runs two assembly lines, A and B. If 95% of line A's products pass inspection, while only 90% of line B's products pass inspection, and 60% of the factory's bike come off assembly line B (the rest off A), find the probability of a bike not passing inspection.

4. A survey has shown that 52% of the women in a certain community work outside the home. Of these women, 64% are married, while 86% of the women who do not work outside the home are married. Find the probabilities that a woman in the community can be categorized as follows:

   (a) A single woman working outside the home.

   (b) Married.

5. In one area, 4% of the population drive luxury cars. However, 17% of the CPAs drive luxury cars. Are the events "person drives a luxury car" and "person is a CPA" independent?

6. On a typical January day in Manhattan the probability of snow is 0.10, the probability of a traffic jam is 0.80, and the probability of snow or a traffic jam is 0.82. Are the events "it snows" and "a traffic jam occurs" independent?

7. A particular rocket assembly depends on two components, A and B. If either is working properly, the assembly will function, whereas if both fail, the assembly will not function. Assume that the functioning of either componenet is independent of the other, that the probability of failure of A is 0.002, and that the probability of failure of B is 0.01.

   (a) What is the probability that both components will fail simultaneously?

   (b) What is the probability that at least one of the components is working properly?

   (c) What is the probability that one component will be functioning properly and one will not?

8. A building contractor buys 70% of his cement from supplier A, and 30% from supplier B. A total of 90% of the bags from A arrive undamaged, while 95% of the bags from B arrive undamaged. Find the probability that a damaged bag came from supplier B.

9. A crate contains 8 basketballs and 5 footballs. A bag contains 6 basketballs and 9 footballs. An experiment consists of selecting a ball at random from the crate and placing it in the bag. A ball is then randomly selected from the bag. If a football is selected from the bag, what is the probability that the transferred ball was a football?

10. The probability that a person with certain symptons has hepatitis is 0.8. The blood test used to confirm this diagnosis gives positive results for 90% of people with the disease and 5% of those without the disease. What is the probability that an individual who has the symptons and who reacts positively to the test actually has hepatitis?

11. The following table shows the proportion of people over 18 who are in various age categories, along with the probabilities that a person in a given age category will vote in a general election.

Age	Percent of Voting Age Population	Probability of a Person of this Age Voting
18 – 21	11.0	0.48
22 – 24	7.6	0.53
25 – 44	37.6	0.68
45 – 64	28.3	0.64
65 or over	15.5	0.74

(a) Find the probability that a voter is in the age category 18 – 21.

(b) Find the probability that a person who did not vote was in the age category 45 – 64.

Section 8.1

- A **random variable** is a rule that assigns a number to each outcome of an experiment.
- A **finite discrete** random variable is one which can only take on a limited number of values that can be listed.
- An **infinite discrete** random variable is one which can take on an unlimited number of values that can be listed in some sort of sequence.
- A random variable is **continuous** if the values it may assume comprise an interval of real numbers.
- We use **histograms** to visually represent probability distributions of random variables.

12. Classify each of the following random variables as finite discrete, infinite discrete, or continuous.

    (a) An experiment consists of testing the bottling accuracy of a 20 ounce coke. Let X represent the actual amount of liquid in a 20 ounce bottle of coke.

    (b) Let Y represent the number of defective shoes made in a Nike factory in a given day.

    (c) An experiment consists of selecting marbles without replacement from a bowl that contains 10 yellow, 13 green, and 8 red marbles. Let Z represent the number of marbles selected until a blue marble is drawn.

    (d) An experiment consists of selecting marbles with replacement from a bowl that contains 10 yellow, 13 green, and 8 red marbles. Let X represent the number of marbles selected until a blue marble is drawn.

    (e) Let Y represent the amount of time (in minutes) that it takes to complete a transaction at an ATM.

13. An experiment consists of flipping a fair coin 5 times. Let the random variable X represent the number of times the coin lands on heads.

(a) Find the probability distribution of X.

(b) Draw the histogram for the random variable X.

(c) Find $P(X > 3)$

14. An experiment consists of randomly selecting a sample of 4 radios out of a bin containing 30 radios, of which 6 are defective. Let the random variable Y represent the number of defectives in the sample. Find the probability distribution of Y.