Fall 2011
Week-in-Review #10
(courtesy: Kendra Kilmer)
(covering Sections 8.5, 8.6, and 5.1)

Sections 8.5 and 8.6

- If \( X \) is a **continuous random variable**, a **probability density function** is defined to represent the probability distribution of \( X \). The curve lies completely above the \( x \)-axis and the total area under this curve is 1.
- A normal random variable is defined by its mean (\( \mu \)) and standard deviation (\( \sigma \)). The probability density function associated with a normal random variable has its peak directly above the mean and is symmetric about a vertical line passing through the mean. The **standard normal variable** \( Z \) has a mean of 0 and a standard deviation of 1.
- To find probabilities associated with a Normal Random Variable, press 2nd VARS and the select option 2:normalcdf. On your homescreen, enter normalcdf(left bound, right bound, mean, standard deviation).
- If you are given a probability and asked to find a bound use 3:invNorm. If you are trying to find a, then \( a = \text{invNorm} \) (total area under the curve to the left of \( a \), mean, standard deviation).

1. Let \( Z \) be the standard normal variable. Find the following:
   - (a) \( P(Z \geq 1.5) \)
   - (b) \( P(Z < 2) \)
   - (c) \( P(-1.5 < Z \leq 2) \)

2. Let \( X \) be a normal random variable with \( \mu = 80 \) and \( \sigma = 5 \). Find the following:
   - (a) \( P(X \leq 70) \)
   - (b) \( P(X > 75) \)
   - (c) \( P(45 \leq X \leq 90) \)

3. Let \( Z \) be the standard normal variable. In each of the following, find \( a \).
   - (a) \( P(Z \leq a) = 0.7524 \)
   - (b) \( P(Z \geq a) = 0.4268 \)
   - (c) \( P(Z \geq -a) = 0.2657 \)
   - (d) \( P(-a \leq Z \leq a) = 0.7587 \)

4. Annual salaries at a local company are normally distributed with a mean of $30,000 and a standard deviation of $5,000. If a person is randomly selected from the company, what is the probability that their annual salary is more than $25,000?

5. Grades on a history exam were normally distributed with a mean of 75 and a standard deviation of 10. If 8% of the students received an A, 15% received a B, and 38% received a C, what is the cut-off for an A, B, and C?

Section 5.1

- **Simple Interest**: \( A = P(1 + rt) \) where \( A \) is the accumulated amount, \( P \) is the principal amount, \( r \) is the annual interest rate as a decimal, and \( t \) is time in years.
- **Compound Interest**: \( A = P \left( 1 + \frac{r}{m} \right)^{mt} \) where \( A, P, r, \) and \( t \) represent the same as above and \( m \) is the number of compounding periods per year.
- **Continuous Compound Interest**: \( A = Pe^{rt} \) where \( A, P, r, \) and \( t \) represent the same as above and \( e \approx 2.718281828 \).
- **TVM Solver**:

\[ N = \text{the total number of compounding periods} \]

\[ I\% = \text{interest rate (as a percentage)} \]
\[ PV = \text{present value (principal amount)} \]
\[ PMT = \text{payment amount (0 if no payments are involved)} \]
\[ FV = \text{future value (accumulated amount)} \]
\[ P/Y = C/Y = \text{the number of compounding periods per year} \]

Move the cursor to the value you are solving for and hit ALPHA and then ENTER. In all of the problems we do make sure that END is highlighted at the bottom of the screen. This represents that payments are received at the end of each period.

- The **Effective Rate of Interest** gives the equivalent interest rate if compounding was only done once a year. Use option C:Eff( on the finance menu. Enter as Eff\((\text{interest rate as a percentage}, \text{number of compounding periods per year})\)

6. How much would you have to deposit in an account earning an annual 4.5% simple interest rate if you wanted to have $1000 after 6 years?
7. At what simple interest rate will an investment triple in 20 years?
8. How much would you have in 8 years if you invest $1,500 into an account earning 5.4% per year compounded quarterly?
9. How long will it take an investment to double if it is earning 3.5% annual interest compounded monthly?
10. What is the future value of $35,000 invested at 6.5% annual interest compounded continuously for 5 years?
11. Which would give you a better return: 3.89% per year compounded daily or 3.91% per year compounded quarterly?
12. What is the effective rate of interest of an account that grows from $1,000 to $2,500 in 10 years if the account pays interest that is compounded monthly?